Name: _____

Grade:_____

School: _____

CONNECTICUT ARML QUALIFICATION TEST
Thursday, March 11, 2021

Pre-test instructions:

- When you receive this test, print it out and staple it. Do not read it.
- Then gather all you will need for the test: pencils, scrap paper. (Graph paper and protractors are not allowed.)
- Put your phone, all calculators, and any other electronic devices apart from your computer on a surface way out of reach. (You will need your phone at the end of the test in order to take images of your scratchwork.)
- Go to the bathroom now, so that you won't need to during the test.
- On your computer, sign on to the Zoom meeting you have been given for this test and position the computer camera on your work area in such a way that your hands can be seen during the test.
- Read the directions below and wait to be told to start the test.

Test directions:

- You will be given 90 minutes in which to answer 20 questions.
- All questions carry equal weight.
- All answers are positive integers.
- Write your work in the space provided. You will be required to submit your work at the end of the test in order to confirm the authenticity of your answers. Your work will not be graded.
- All the usual rules for testing apply to this test, including the fact that no communication of any sort with any person is allowed, except with a proctor of this test. (You will be asked to sign a pledge at the end of the test.)
- Books, class notes, etc. are **<u>not</u>** allowed.
- Calculators and the Internet are **<u>not</u>** allowed.

1. Solve for x: $2 \log \left(\frac{1}{2}x\right) - \log(2x) = 0$

<div align="right">Answer:_____</div>

2. Suppose that $f(1) = 1$ and, for all $n > 1$, $f(n) = (n-1) \cdot f(n-1)$. Find $f(4)$.

<div align="right">Answer:_____</div>

3. What is the last digit of 7^{77} ?

Answer:_____

4. How many integers are in the solution set of the inequality $2 \le |x - 5| < 7$?

Answer:_____

5. A tetrahedral die (with faces numbered 1 through 4) and an octahedral die (with faces numbered 1 through 8) are rolled. The probability that the sum of the numbers rolled is divisible by 3 is $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

Answer:_____

6. Find the value of the term that does not contain an x in the expansion of $\left(x^{2/3} - \frac{2}{x^{1/3}}\right)^9$.

Answer:_____

7. Find the number of scalene triangles with integer side-lengths, nonzero area, and perimeter less than 13.

Answer:_____

8. The areas of three faces of a rectangular solid are 3, 6, and 8. Find the volume of the rectangular solid.

Answer:_____

9. In the land of Joy, numbers are written in base *J*. Bob bought a computer for 560 monetary units (m.u.). He gave the clerk a 1000 m.u. bill and received 770 m.u. in change. What is the base *J* ?

Answer:_____

10. In 2020, February 29th was a Saturday. In what year will February 29th next be a Saturday?

Answer:_____

11. Rectangle *ABCD* has a width of 15 and a length of 20. Diagonal *AC* is divided into four equal segments by successive points *E*, *F*, and *G*. Find the area of triangle *BEG*.

Answer:_____

12. From six girls and three boys, in how many different ways can a math team of five be chosen if the team is to contain at least three girls?

Answer:_____

13. Isosceles triangle ABC has $AB = 6$, $BC = AC$, and $m\angle C = 30°$. The area of triangle ABC is $a + b\sqrt{c}$, where a and b are integers and c is a positive integer that is not divisible by the square of any prime number. Find $a + b + c$.

Answer:_____

14. Let S be the set of all arrangements of the digits 2, 3, 4, 5, 8 such that the five-digit number formed is divisible by 55. Find the second-largest number in S.

Answer:_____

15. Find the sum of all positive integer values of k for which the expression $x^2 + kx + k + 11$ can be written as the product of two linear binomials with integer coefficients.

Answer:_____

16. Lines l_1 and l_2 have distinct positive slopes and the measure of the acute angle between the two lines is 45°. The slope of l_1 is $\frac{2}{3}$. Find the slope of l_2.

Answer:_____

17. In a geometric sequence of real numbers, the sum of the first two terms is 5 and the sum of the next four terms is 100. Find the sum of the first four terms of the sequence.

Answer:_____

18. Find the angle A, $0 < A < 360°$, such that $\cot A - \csc A = \sqrt{3}$. (Do not include a unit in your answer.)

Answer:_____

19. Evaluate:

$$\cos^2\left(\frac{\pi}{12}\right) + \cos^2\left(\frac{2\pi}{12}\right) + \cos^2\left(\frac{3\pi}{12}\right) + \cdots + \cos^2\left(\frac{11\pi}{12}\right)$$

Answer:_____

20. *ABC* is a triangle of nonzero area. $A = (0,0)$, $B = (36,15)$, and both of the coordinates of C are integers. The minimum possible area of triangle *ABC* is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

Answer:_____

END OF TEST

Post-test directions. You have 10 minutes to complete steps 1–3 below.

1. Enter your answers on the Google Form provided in the Zoom chat and submit the form. (You have 3½ minutes to do this.)

2. Please complete the following pledge by crossing out the words that do not apply, and add your signature.

   I <u>did / did not</u> receive help from any person or source on this test.
   I <u>did / did not</u> use a calculator and/or the Internet on this test.
   I <u>did / did not</u> abide by exam rules exactly as is expected in a classroom.

   Signed:_____

3. Get your phone and take images of all pages, including the front page (which includes your name) and this page (which includes your signed pledge.) Preferably, please use a scanning app and submit the images as a single pdf. Send your pdf (or photos) by email to CTARMLTeam@gmail.com. Please include your name in the subject line.

4. Please remain on the Zoom meeting until you are told that you may leave.