Name: _____

Grade:_____

School: _____

CONNECTICUT ARML QUALIFICATION TEST
Thursday, March 10, 2022

Pre-test instructions:

- When you receive this test, print it out and staple it. Do not read it.
- Then gather all you will need for the test: pencils, scrap paper. (Graph paper and protractors are not allowed.)
- Put your phone, all calculators, and any other electronic devices apart from your computer on a surface way out of reach. (You will need your phone at the end of the test in order to take images of your scratchwork.)
- Go to the bathroom now, so that you won't need to during the test.
- Your computer should still be signed on to the Zoom meeting you have been given for this test. Position your computer camera on your desk about 20 inches to the side of the place you will be working. Your entire work area should be visible to the proctors, and it must **not** be possible for other participants to read what you are writing.
- Read the directions below and wait to be told to start the test.

Test directions:

- You will be given 1 hour 45 minutes in which to answer 25 questions.
- All questions carry equal weight.
- All answers are positive integers.
- Write your work in the space provided. You will be required to submit your work at the end of the test in order to confirm the authenticity of your answers. This scratchwork will not be graded.
- All the usual rules for testing apply to this test, including the fact that no communication of any sort with any person is allowed, except with a proctor of this test. (You will be asked to sign a pledge at the end of the test.)
- Books, class notes, etc. are **not** allowed.
- Calculators and the Internet are **not** allowed.

1. The sum of two prime numbers is 75. Find the product of these two prime numbers.
   [Answer: 146]

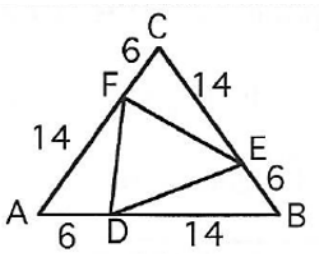
2. For what positive number-base *b* is it true that $(23_b)^2 = 562_b$?
   [Answer: 7]

3. Suppose that g is a function with $g(1) = 1$, $g(2) = 3$, and with the property that, for all integers *n*, $g(n) = g(n-1) - g(n-2)$. Find $g(38)$.
   [Answer: 3]

4. Find the sum of the squares of the solutions of the equation $x(|x| - 5) = -6$.
   [Answer: 49]

5. The digits 2, 2, 3, 4, 5 will be arranged to form a five-digit positive integer, and the two 2s will not be placed next to each other. How many such five-digit integers are possible?
   [Answer: 36]

6. In the diagram below, points *D*, *E*, *F* lie on line segments *AB*, *BC*, *CA*, respectively. The ratio of the area of triangle *DEF* to the area of triangle *ABC* is $\frac{a}{b}$, where *a* and *b* are relatively prime positive integers. Find $a + b$.



   [Answer: 137]

7. Compute $99^5 + 5 \cdot 99^4 + 10 \cdot 99^3 + 10 \cdot 99^2 + 5 \cdot 99$.
   [Answer: 9999999999]

8. The expression $x^4 - 11x^2 + 49$ can be factored uniquely in the form $(x^2 + px + q)(x^2 + rx + s)$, where *p*, *q*, *r*, *s* are integers. Find $p^2 + q^2 + r^2 + s^2$.
   [Answer: 148]

9. Evaluate: $\log_2(3) \cdot \log_3(4) \cdot \log_4(5) \cdot \cdots \cdot \log_{31}(32)$.
   [Answer: 5]

10. In square *PQRS*, *T* is the midpoint of side *QR*. The line through *S* perpendicular to line *PT* intersects line *PT* at *U*. If *PQ* = 2, the distance *SU* is $\frac{a\sqrt{b}}{c}$, where *a*, *b*, *c* are integers, *a* and *c* are relatively prime, and *b* is not divisible by the square of any prime number. Find $a + b + c$.
    [Answer: 14]

11. Find the sum of the squares of all values of the constant *p* such that the vertex of the parabola $y = px^2 + (5p + 3)x + (6p + 5)$ lies on the *x*-axis.
    [Answer: 82]

12. Let *a* and *b* be positive integers, and suppose that the real part of $(a + bi)^3$ is −9. Find $a + b$.
[Answer: 5]

13. Let $S = \{1, 2, 3, 4, \ldots, 19, 20\}$. A subset of *S* consisting of *m* consecutive integers will be selected, where $m \geq 1$. Find the number of possible such subsets.
[Answer: 210]

14. Let g be that function such that, for all values of *x*, $g(x) + 3g(6 - x) = x^2 + 1$. Then $g(2) = \frac{a}{b}$, where *a* and *b* are relatively prime integers and $b > 0$. Find $a + b$.
[Answer: 27]

15. A bag contains one blue disk and one yellow disk. A second bag contains one blue disk and three yellow disks. For either of the bags, when a disk is picked from the bag each disk in the bag is equally likely to be picked.

    One disk is picked from each bag and placed on a shelf. If there is exactly one blue disk on the shelf, a further disk is picked from the bag that still contains a blue disk, and the selected disk is placed on the shelf. The probability that both blue disks are placed on the shelf during this game is $\frac{a}{b}$, where *a* and *b* are relatively prime positive integers. Find $a + b$.
[Answer: 11]

16. Let *PQR* be a triangle with sides of length 3, 4, and 5. Let *X* be any point. The minimum possible value of $PX^2 + QX^2 + RX^2$ is $\frac{a}{b}$, where *a* and *b* are relatively prime positive integers. Find $a + b$.
[Answer: 53]

17. In a rhombus *PQRS*, suppose that each side has length *k*, $\cos(\text{m}\angle P) = \frac{2}{3}$, and the radius of the circle that passes through the points *P*, *Q*, and *S* is 1. Then $k^2 = \frac{a}{b}$, where *a* and *b* are relatively prime positive integers. Find $a + b$.
[Answer: 13]

18. The sum of the four solutions, real and/or complex, of the equation

$$\frac{3x^2 - 3x + 4}{x^2 + 5x - 4} = \frac{x^2 - 6x - 9}{x^2 + x + 3}$$

    is $(-1)^n \cdot \frac{a}{b}$, where *a* and *b* are relatively prime positive integers and *n* is 0 or 1. Find $n + a + b$.
[Answer: 4]

19. For how many ordered pairs (a, b) of positive integers is $15a + 22b = 2201$?
[Answer: 7]

20. Suppose that $x > 0$, $y > 0$, and $\log_9(x) = \log_{12} y = \log_{16}(x + y)$. Then $\frac{y}{x} = \frac{a+\sqrt{b}}{c}$, where *a*, *b*, *c* are positive integers and *a*, *c* are relatively prime. Find $a + b + c$.
[Answer: 8]

21. A pyramid has a square base of side-length 640. The base lies in a horizontal plane and the height of the vertex of the pyramid above the base is 1024. The pyramid is now cut by a horizontal plane at height *h* above the base. The part of the pyramid above this plane is a smaller pyramid. For how many integer values of *h* with $1 \le h \le 1023$ is the volume of the smaller pyramid an integer?
[Answer: 85]

22. Let α be an angle such that $\sin\alpha, \sin 2\alpha, \sin 4\alpha$ is an arithmetic sequence with nonzero common difference. Then $\cos\alpha - \cos^3\alpha = \frac{a}{b}$, where *a* and *b* are relatively prime integers and $b > 0$. Find $a + b$.
[Answer: 9]

23. The *period* of a sequence $a_0, a_1, a_2, a_3, \ldots$ is the smallest positive integer *k* such that $a_{n+k} = a_n$ for all $n \ge 0$. A sequence is given by $a_0 = \tan 4°$, $a_n = \frac{2a_{n-1}}{1-a_{n-1}^2}$ (for $n > 0$). Find the period of this sequence.
[Answer: 12]

24. Find the sum of all real values of *x* that satisfy the equation $\sqrt[3]{5x + \sqrt[3]{5x + 11}} = 11$.
[Answer: 264]

25. For how many integers *n*, $1 \le n \le 100$, is $x^{2n} + 1 + (x+1)^{2n}$ divisible by $x^2 + x + 1$?
[Answer: 67]

END OF TEST

Post-test directions. You have 10 minutes to complete steps 1–3 below.

1. Enter your answers on the Google Form provided in the Zoom chat. Check that you have typed your answers correctly, and then submit the form. (You have 3½ minutes to do this.)

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2. Please complete the following pledge by crossing out the words that do not apply, and add your signature.

   I did / did not abide by exam rules exactly as is expected in a classroom.
   I did / did not complete this test without the help of any person or source.
   I did / did not complete this test without the use of a calculator and/or the Internet.

   Signed:_____

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3. Get your phone, and using the Adobe Scan app, create a PDF consisting of all the pages of this test, including the front page (which includes your name) and this page (which includes your signed pledge). For this document, select share, email, and send the link by email to CTARMLTeam@gmail.com. Please include your name in the subject line.

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4. Please remain on the Zoom meeting until you are told that you may leave.
