NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS - 2023

Round 1: Arithmetic and Number Theory

1. ____________________

2. ____________________

3. $____________________

1. The sum of $0.\overline{12}$ and $0.\overline{252}$ can be written as $\frac{a}{b}$, where *a* and *b* are relatively prime. Compute the sum $a+b$.

2. *N* divided by 4/7, *N* divided by 3/14, and *N* divided by 5/21 all produce integer quotients. Determine the smallest possible positive value of *N*.

3. Based on information from www.fueleconomy.gov, a certain plug-in hybrid uses 25 kwh of electricity to go 100 miles. The same car gets 25 mpg using regular fuel. Suppose electricity costs \$0.14 per kwh and gas costs \$2.70 per gallon. In driving 10,000 miles the hybrid is solely powered by gas for 1000 miles and solely powered by electricity for 9000 miles. Let *N* be the cost of using the hybrid on that trip. Let *M* be the cost of a 10,000 mile trip using a car solely powered by gas that gets 30 mpg. Compute $M-N$.

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Round 2: Algebra 1

1. ______________________

2. ______________________

3. ______________________

1. Solve: $\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{3}} < \sqrt{27}$.

2. Compute the value of $x + y + z$, given the system

$$\begin{cases} x + y - z = -5 \\ xy = z \\ y - z = -9 \end{cases} .$$

3. Compute all values of k for which $x^2 + 3x - k = 0$ and $x^2 - 5x - 5k = 0$ have a common solution.

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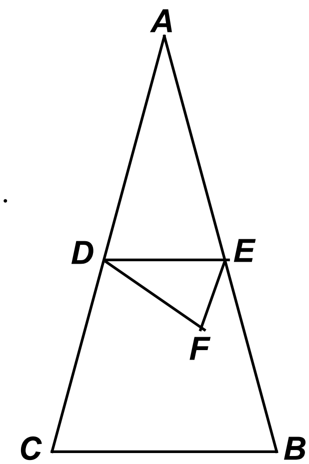
Round 3: Geometry

1. ______________________

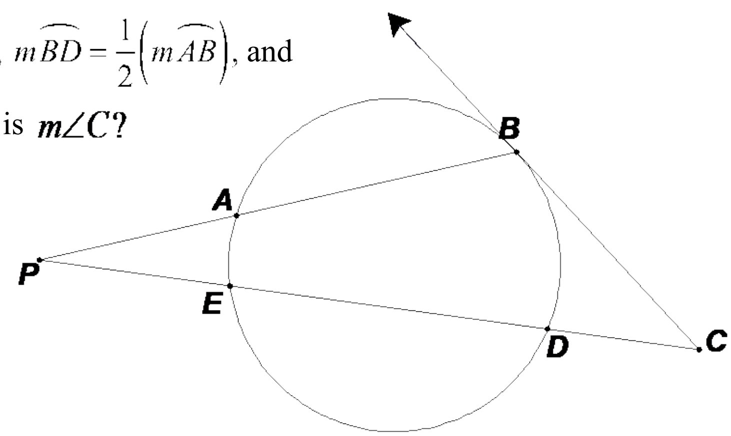
2. ______________________°

3. ______________________



1. In triangle ABC, $AB = AC = 2$ and $BC = 1$.
D and E are midpoints of $\overline{AC}$ and $\overline{AB}$, respectively, and
F lies in the interior of ΔABC such that $DE = DF$ and $\angle EDF \cong \angle A$.
Compute the area of ΔDEF.

2. If $m\overset{\frown}{AB} = 4\left(m\overset{\frown}{AE}\right)$, $m\overset{\frown}{ED} = 5\left(m\overset{\frown}{AE}\right)$, $m\overset{\frown}{BD} = \frac{1}{2}\left(m\overset{\frown}{AB}\right)$, and
$\overline{TC}$ is tangent to the circle at B, what is $m\angle C$?



3. Let $A_2A_4A_6A_8$ be a square of side 1, let P be the point where the diagonals of the square intersect, and let A_1, A_3, A_5, and A_7 be the midpoints of A_2A_8, A_2A_4, A_4A_6, A_6A_8, respectively. From P, segments are drawn through each A_i to a point B_i such that $PA_i \cdot PB_i = 1$.
Determine the exact area of the region bounded by
$\overline{B_1B_2} \cup \overline{B_2B_3} \cup \overline{B_3B_4} \cup \overline{B_4B_5} \cup \overline{B_5B_6} \cup \overline{B_6B_7} \cup \overline{B_7B_8} \cup \overline{B_8B_1}$.

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Round 4: Algebra 2

1. ________________________

2. ________________________

3. ________________________

1. Compute the value of x satisfying $2^{\log_x 9} = \frac{1}{4}$.

2. Given: $f(x) = (3+2x)(1-4x)$
Compute the minimum positive integer value of c for which the maximum value of the function $g(x) = f\left(\frac{x+1}{2}\right) - c$ is negative.

3. As he wrote his last contest, Don Barry was reflecting on the 37 years that he'd written problems for MAML and NEAML. The phrase ITHASBEENAPLEASURE came to mind. He then wondered what the probability would be that the phrase HASBEEN would appear somewhere in the string, if the letters were arranged at random. Compute the answer to his question.

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Round 5: Analytic Geometry

1. ______________________

2. ______________________

3. ______________________

1. A line passing through the origin passes below $A(1,4)$ and above $B(7,2)$. If the vertical distance of the line below *A* is the same as the vertical distance above *B*, what is the slope of the line?

2. Let $f(x)=mx+k$, where both *m* and *k* are positive integers. If f **and** f^{-1} intersect at a point where $x=-5$, what is the largest possible value of *k* such that *m* is less than 100?

3. Given: $f(x)=-(x+1)(x-5)$

   A and *B* are the points where the graph of $y=f(x)$ intersects the *x*-axis.

   P lies on the graph of $y=f(x)$, where $f(x)>0$.

   ΔAPB is a right triangle.

   Determine all possible *x*-coordinates of *P*.

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Round 6: Trig and Complex Numbers

1. ____________________ °

2. ______________________

3. ______________________

1. For $0° < \theta < 32°$, if $\cos\theta = \cos(11\theta)$, what is the value of θ?

2. Solve $\cos(2x) - \tan(x) = 1$ for $0 \le x < 2\pi$.

3. Let $A = \frac{1}{2}\left(\cos 50° + i\sin 50°\right)$ and $B = 2\left(\cos 18° + i\sin 18°\right)$. Compute the area of the triangle in the complex plane whose vertices are the origin, A^3, and B^5.

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Team Round - Place all answers on the team answer sheet.

1. Let M_0 be a "4-digit" positive integer, from 0001 to 9999, inclusive.
   Create a new "4-digit" positive integer $M_1 = \underline{A}\,\underline{B}\,\underline{C}\,\underline{D}$, where
   A denotes the number of odd digits in each "4-digit" positive integer.
   B denotes the number of even digits in each "4-digit" positive integer.
   $C = |A - B|$
   $D = A \cdot B$.
   An infinite sequence of integers $M_0, M_1, M_2, M_3, \ldots$ is created using this procedure.
   Compute the sum of all distinct possible values of M_{2023}.

2. Consider the following three listings of the integers n from 1 to 25, inclusive, where duplications are allowed:
   D_1: each value of n occurs n times
   D_2: each value of n occurs 2^{n-1} times
   D_3: each value of n occurs $n - \phi(n)$ times, where $\phi(n)$ denotes the number of integers between 1 and n, inclusive, that are relatively prime to n.
   m_1, m_2, and m_3 are the mean, median, and mode, respectively, of D_1, D_2, and D_3.
   Compute the ordered triple (m_1, m_2, m_3).

3. The number of square units in the lateral surface area of a right circular cone whose radius is r and whose height is h is equal to the number of cubic inches in its volume. Compute the smallest integer value of h^2 that makes r an integer.

4. Marty Badoian designed a format for a math competition. Each team of 10 had to consist of 2 freshmen (A,B), 2 sophomores (C,D), 3 juniors (E,F,G), and 3 seniors (H,I,J). The contest had 3 rounds: geometry, algebra, and trig. Each team had to enter a group of 4 students in each round, each group of 4 had to have one member from each class, and no member could appear in more than 2 rounds. How many different team rosters could a school submit for this three-round contest?

	1	2	3
A			
B			
C			
D			
E			
F			
G			
H			
I			
J			

5. Compute the minimum distance between the graphs of $y = -x^2 + 4x - 3$ and $4x + 2y - 17 = 0$.

6. Given: $\theta = \text{Tan}^{-1}\left((4x-1)(3x+2)\right)$
   Determine all values of x for which $0 < 2\theta < \dfrac{\pi}{2}$.

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Answer Sheet

Round 1

1. 559
2. $\frac{60}{7}$
3. 477

Round 2

1. $x > \frac{3}{100}$
2. 19
3. $k = 0, 10$

Round 3

1. $\frac{\sqrt{15}}{64}$
2. 45
3. 8

Round 4

1. $\frac{1}{3}$
2. 7
3. $\frac{1}{185,640}$

Round 5

1. $\frac{3}{4}$
2. 490
3. $2 \pm 2\sqrt{2}$

Round 6

1. 30
2. $0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}$
3. $\sqrt{3}$

Team

1. 3563
2. $(17, 25, 24)$
3. 12
4. 20,736
5. $\frac{\sqrt{5}}{2}$
6. $-\frac{3}{4} < x < -\frac{2}{3}$ or $\frac{1}{4} < x < \frac{1}{3}$