

CT ARML Team, 2023

Team Selection Test 2

1. Andy is taking free throws. His *shooting accuracy* is the percentage of throws on which he is successful. After taking n free throws his shooting accuracy is 65%. He then makes k successful throws, and his shooting accuracy after $n + k$ throws is 70%. Compute the minimum possible value of k .
[Answer: 10]
2. A particle starts moving on the number line at time $t = 0$. Its position at time t is $x = (t - 2022)^2 - 2022(t - 2022) - 2023$. Compute the number of positive integer values of t at which the particle lies in the negative half of the number line (strictly to the left of 0).
[Answer: 2023]
3. For each positive integer n , let $x_n + iy_n = (1 + i\sqrt{3})^n$, where x_n and y_n are real. Suppose that $x_{19}y_{91} + x_{91}y_{19} = 2^k\sqrt{3}$. Compute k .
[Answer: 109]
4. Let Ω be a semicircle with diameter $AB = 10$. There exist points C and D on Ω (meaning that C and D lie on the arc \widehat{AB}) such that \overline{AC} and \overline{BD} intersect at a point E in the interior of the semicircle, with $AE = 6$ and $CE = 2$. Then $AD^2 = \frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.
[Answer: 167]
5. Find the number of four-digit numbers $\underline{A}\underline{B}\underline{C}\underline{D}$ with distinct, nonzero digits such that $A < B$ and $C < D$.
[Answer: 756]
6. For real numbers x , let $f(x) = 16x^3 - 21x$. Given that θ is an angle satisfying $f(\sin \theta) = f(\cos \theta)$, the sum of all possible values of $\tan^2 \theta$ is $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.
[Answer: 256]

7. Let f and g be quadratic polynomials. Suppose that f has zeros 2 and 7, g has zeros 1 and 8, and $f - g$ has zeros 4 and 5. Compute the product of the zeros of the polynomial $f + g$.

[Answer: 12]

8. The base-10 fraction $\frac{20}{23}$ has period a when written (as a repeating decimal) in base 2023. Compute the value of a .
(Note: The *period* of a repeating decimal is the number of digits after which the pattern repeats. For example, the period of $0.476476476\dots$ is 3.)

[Answer: 2]

9. Find the greatest possible integer n such that there is a unique positive integer m that satisfies

$$\frac{1}{10} \leq \frac{m}{n} \leq \frac{1}{9}.$$

[Answer: 161]

10. James has 16 identical apples. James will distribute these apples among his six friends: Albert, Bevin, Camila, Davon, Emma, and Frank. Each of the first three of these people will receive a number of apples that is a multiple of 3 (possibly 0), and each of the last three will receive a number of apples that is not a multiple of 3. Compute the number of possible ways in which James can distribute his apples.

[Answer: 378]