CONNECTICUT STATE ASSOCIATION OF MATH LEAGUES
State Math Match, 2024

Please write your answers on the answer sheet provided.

Round 1: Arithmetic and Number Theory

1-1 For any positive integer n, let $p(n)$ be the number of distinct prime numbers that divide n. For example, $p(12) = 2$, since the only prime numbers that divide 12 are 2 and 3. Find $p(2024^2)$.

1-2 Annabel is on an 8-day cycle in which she works 7 consecutive days, then gets 1 rest day. Boris is on a 6-day cycle in which he works 5 consecutive days, then gets 1 rest day. There are 366 days this year. If they start working on January 1st, how many rest days will they have together this year?

1-3 Let $S = \{2^0, 2^1, 2^2, \ldots, 2^{n-1}\}$, where $30 \leq n \leq 35$. Let T be a subset of S such that the sum of the elements of T is exactly $\frac{1}{5}$ of the sum of the elements of S. How many numbers are there in the set T?

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Round 2: Algebra I

2-1 Find the area of the region bounded by the lines $y = 4 - x$, $y = 3x - 8$, and the y-axis.

2-2 There are 34 houses in a street. Some of the houses have solar panels, the rest do not. If 10 of the houses that don't have solar panels now install solar panels, the number of houses that now have solar panels is equal to the number that originally did not. How many houses originally had solar panels?

2-3 The sum of all real solutions of the equation

$$\frac{8x^2 + x + 9}{5x^2 + 8x + 8} = \frac{9x^2 + 2x + 10}{6x^2 + 9x + 9}$$

is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

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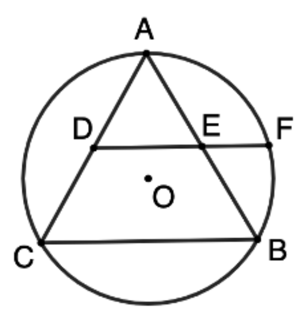
Round 3: Geometry

3-1 For a particular cube, the sum of the lengths of its edges in centimeters is numerically equal to its volume in cubic centimeters. Find the surface area of the cube in square centimeters. (Do not include a unit in your answer.)

3-2 In an isosceles trapezoid, the length of each leg is 3, the length of each diagonal is 7, and the length of the shorter base is 5. Find the length of the longer base.

3-3 The diagram below shows an equilateral triangle ABC inscribed in a circle whose center is O. The points D and E are the midpoints of sides $\overline{AC}$ and $\overline{AB}$, respectively. Line $\overline{DE}$ intersects the circle at point F, with E between D and F. It follows that $\frac{DE}{EF} = \frac{a+\sqrt{b}}{c}$, where a, b, and c are positive integers and b is not divisible by the square of any prime number. Find $a + b + c$.



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Round 4: Algebra II

4-1 Let $f(x) = x^2 - 2x + 11$ and $g(x) = \sqrt[3]{x+1}$. Find $f(g(7)) + g(f(-3))$.

4-2 An infinite geometric series has a positive common ratio. The sum of the first two terms is 5 and the sum of the entire series is 9. The common ratio can be written as $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

4-3 Let $P(x)$ be a polynomial whose degree is 56 and suppose that $P(n) = \frac{1}{n}$ for $n = 1, 2, 3, \ldots, 57$. Then $P(58) = \frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

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Round 5: Analytic Geometry

5-1 Suppose that the point $(k+3, k-1)$ lies on the curve with equation $2y^2 - x = 6$. The larger of the two possible values of k is $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a+b$.

5-2 The straight line that passes through the vertices of the parabolas $y = 2x^2 - 8x + 11$ and $y = 3x^2 - 6x + 1$ intersects the y-axis at the point $(0, k)$. Find $|k|$.

5-3 Find the number of intersection points of the graphs of $(x - \lfloor x \rfloor)^2 + y^2 = x - \lfloor x \rfloor$ and $y = \frac{1}{5}x$.
(Note: $\lfloor x \rfloor$, known as the *floor* of x, is the greatest integer less than or equal to x.)

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Round 6: Trigonometry and Complex Numbers

6-1 Let $\sin 143° \cos 23° - \cos 143° \sin 23° = k$. Then $k^2 = \frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

6-2 Suppose that $0 < \theta < \frac{\pi}{2}$ and $\cos\theta - \sin\theta = \frac{1}{2}$. Then $\cos\theta = \frac{a+\sqrt{b}}{c}$, where a, b, and c are positive integers and b is not divisible by the square of any prime number. Find $a + b + c$.

6-3 Let z be a complex number with the property that $z + \frac{400}{z}$ is a real number and $11 < z + \frac{400}{z} < 31$. Assume, further, that the real part of z is a positive integer and the imaginary part of z is positive. Find the sum of all possible values of the real part of z.

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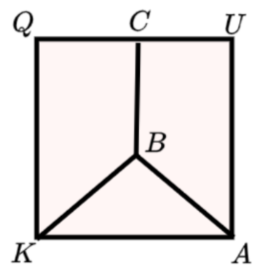
Team Round

T-1 Suppose that a and b are positive integers and $a^2 - b^2 = 144$. What is the largest possible value for the greatest common divisor of a and b?

T-2 Suppose that $22x + 23y = 16$ and $23x + 22y = 29$. Find $x^2 + y^2$.

T-3 In square $QUAK$, shown in the diagram below, $AU = 2$, $\overline{CB} \perp \overline{QU}$, and $BC = BK = AB$.



It follows that $BC = \frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

T-4 The sequence $a_1, a_2, a_3, \ldots$ is defined by $a_1 = 1$, and, for $n > 1$, $a_n = a_{\sqrt{n}}$ if n is a perfect square, otherwise $a_n = a_{n-1} + 1$. Find the smallest value of n for which $a_n = 103$.

T-5 Find the area enclosed by the graph of $y = |x| + |x - 2|$ and the line $y = 8$.

T-6 Let $z = -1 + i\sqrt{3}$ and $w = -\sqrt{3} + i$. For how many integer values of n, with $2 \leq n \leq 100$, do w and one of the complex n^{th} roots of z have the same argument? (We assume that all arguments are written in the range $0 \leq \theta < 2\pi$.)

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Answers

Round 1

1-1	3
1-2	15
1-3	16

Round 2

2-1	18
2-2	12
2-3	10

Round 3

3-1	72
3-2	8
3-3	8

Round 4

4-1	14
4-2	5
4-3	30

Round 5

5-1	9
5-2	7
5-3	11

Round 6

6-1	7
6-2	12
6-3	105

Team Round

T-1	4
T-2	85
T-3	9
T-4	2115
T-5	30
T-6	8