1. ________________

2. ________________

3. ________________

2024 NEAML

Round 1. -Arithmetic and Number Theory

1. Compute the ordered pair of positive integers (x, y) with $10 \leq x \leq 20$ and $10 \leq y \leq 20$ for which $63x - 44y = 8$.

2. Compute the number of positive integers less than 2022 that can be expressed as the difference of the squares of two consecutive integers.

3. Three positive integers are in increasing arithmetic progression. If the middle integer is decreased by 40, then the new three integers are in geometric progression. If instead the third integer is increased by 320, then the new three integers are in geometric progression. Compute the first integer.

1.______________________

2.______________________

3.______________________

Round 2 - Algebra 1

1. Dora works at twice Diego's pace. If Diego worked three more hours than it would take Dora to do a job, then Diego would finish 75% of the job. Compute the number of hours it would take Dora to do the job.

2. Compute all x such that $(2x-3)^2+(3x-5)^2=(5x-8)^2$.

3. Compute $\dfrac{2023^3+9\cdot 2023^2+23\cdot 2023+15}{2023^3-1992\cdot 2023^2-15985\cdot 2023-30000}$.

1.______________________________

2.______________________________

3.______________________________

Round 3 - Geometry

1. In a field, a square grassy area 100 feet by 100 feet is surrounded by a fence. Two horses walk on different paths (one inside the fenced area and one outside the fenced area), each staying 10 feet from the fence at all times. Given that the greatest possible distance between the horses is M feet, compute the integer closest to M.

2. A right prism has regular hexagons with side length $\sqrt{2023}$ for its bases and six congruent rectangles of area 4046 each for its lateral faces. There are two noncongruent space diagonals in the prism, and these space diagonals have lengths a and b. Compute $|a^2 - b^2|$.

3. In $\triangle TRI$, $TR = 20$, $TI = 18$, and $RI = 12$. Point A is on $\overline{TI}$ such that $\overline{RA}$ bisects $\angle TRI$. Point N is on $\overline{RA}$ such that $\overline{IN}$ bisects $\angle TIR$. Given that $\dfrac{RN}{NA}$ is $\dfrac{p}{q}$ where p and q are whole numbers and the fraction is in lowest terms, compute $p+q$.

1.______________________________

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3.______________________________

Round 4 - Algebra 2

1. Compute x such that $\sqrt{x-3}+x=15$.

2. Suppose that m and n are positive numbers such that $\dfrac{(m+n)^3}{m^3+n^3}=\dfrac{14}{5}$. Compute $\dfrac{m^2+n^2}{mn}$.

3. Given that $x^2+3x+9=0$, compute the value of x^6.

1.________________

2.________________

3.________________

Round 5 - Analytic Geometry

1. In the coordinate plane, the image of the point $(20, 21)$ after a reflection in the line $y = x$ is (A, B). The image of (A, B) after a reflection in the line $y = 2x$ is (C, D). The image of (C, D) after a reflection in the line $y = -\frac{1}{2}x$ is (E, F). Compute $E + F$.

2. Octagon $MOUSEPAD$ has vertices $M(0, 4)$, $O(0, 0)$, $U(7, 0)$, $S(7, 1)$, $E(2, 1)$, $P(2, 3)$, $A(1, 3)$, and $D(1, 4)$. The line $y = kx$ divides $MOUSEPAD$ into two regions of equal area. Compute k.

3. The points $P(14, 15)$ and $Q(20, 23)$ each lie on two distinct circles with radius 13. Compute the distance between the centers of the two circles.

1.____________________

2.____________________

3.____________________

Round 6 - Trig and Complex Numbers

1. Compute the least positive integer N for which $\cos(12N)^\circ$ and $\sin(5N)^\circ$ are both negative.

2. Let $i = \sqrt{-1}$. Suppose that $z = 5 + bi$ for some complex number z and some positive integer b. Given that the imaginary part of z^3 is 8 more than the imaginary part of z^2, compute b.

3. Suppose A and B are first-quadrant angles with $\cos A = \frac{3}{5}$ and $\cos(A + B) = \frac{13}{25}$. Given that $\cos B$ can be expressed in simplest form as $\dfrac{K + 8\sqrt{M}}{N}$ where K, M, and N are whole numbers, compute $K + M + N$.

NEAML TEAM ROUND

1.____________ 2.____________ 3.____________

4.____________ 5.____________ 6.____________

1. Suppose that a, b, and c are real numbers such that a is 60% larger than c and b is 20% larger than c. Given that a is $p\%$ larger than b, compute the integer closest to p.

2. The numbers g_1, g_2, and g_3 with $g_1 < g_2 < g_3$ are in geometric sequence. Given that $g_1 + g_2 = 60$ and $g_3 - g_2 = 56$, compute g_2.

3. Compute the least positive integer n such that $24 \cdot n$ has exactly 24 positive integer divisors.

4. Compute the number of integers N, with $1 \leq N \leq 2023$, that contain at least one digit that is a 0 or a 2 or a 3.

5. Sophia is graphing $f(x) = ax^2 + bx + c$. She graphs five points whose x-coordinates are five consecutive integers. Her y-coordinates are 15, 23, 37, 51, and 71. Emma points out that one of Sophia's y-coordinates is incorrect and that it should be m. Compute m.

6. Compute the area bounded below by the graph of $y = |x - 1| + |2x - 4|$ and above by the graph of $y = 5$.

2024 NEAML

ANSWERS

Round 1

1. $(12, 17)$
2. 1011
3. 20

Round 2.

1. 6
2. $\frac{3}{2}$ and $\frac{5}{3}$ (*need both*)
3. 88

Round 3.

1. 137
2. 2023
3. 25

Round 4.

1. 12
2. $\frac{8}{3}$
3. 729

Round 5.

1. -41
2. $\frac{1}{2}$
3. 24

Round 6.

1. 38
2. 8
3. 278

TEAM ROUND

1. 33
2. 42
3. 15
4. 1281
5. 35
6. $\frac{23}{3}$ or $7\frac{2}{3}$