CONNECTICUT ARML QUALIFICATION TEST
Thursday, March 6, 2025

1. There are 10 delegates at a conference: 6 from Party A and 4 from Party B. A committee of 6 delegates is to be formed such that, on the committee, the number of delegates from Party A is greater than the number of delegates from Party B. In how many ways can this be done?
[Answer: 115]

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2. Find the largest positive integer n such that $30!$ is divisible by 2^n.
[Answer: 26]

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3. Let α, β, and γ be the solutions of the equation $2x^3 + 3x^2 - 5x - 4 = 0$. Then $\alpha^2 + \beta^2 + \gamma^2 = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
[Answer: 33]

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4. Four circles of radius 4 are internally tangent to a circle of radius r, with each of the smaller circles externally tangent to two others. Then $r = a + b\sqrt{c}$, where a, b, and c are positive integers and c is not divisible by the square of any prime number. Find $a + b + c$.
[Answer: 10]

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5. Find the number of values of x with $0 < x < 2\pi$ such that

$$\left(4^{\cos 2x}\right)^{\sin 2x} = 2$$

[Answer: 4]

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6. Let A, B, C, and D be positive integers (not necessarily distinct) such that $A^2 + B^2 = 20$ and $C^2 - D^2 = 24$. Find the greatest possible value for the sum $A + B + C + D$.
[Answer: 18]

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7. Let f be a function with the property that, for any real x, $xf(x) + f(x+2) = x^2$. Find $f(8)$.
[Answer: 36]

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8. In the complex plane, let z and w be numbers satisfying $z^6 = 1$ and $w^4 = -1$. Given that 0, z, w, and $z + w$ form a quadrilateral with nonzero area, the minimum possible area of the quadrilateral can be expressed as $\frac{\sqrt{a}-\sqrt{b}}{c}$, where a, b, and c are positive integers, and neither a nor b is divisible by the square of any prime number. Find $a + b + c$.
[Answer: 12]

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9. Point D lies inside triangle ABC so that $AD = 8$, $BD = 5$, and $m\angle ADC = m\angle ADB = m\angle CDB = 120°$.
Given that $\angle ABC$ is a right angle, find the length CD.
[Answer: 30]

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10. Let n be a positive integer. When n dice are rolled, the nonzero probability of obtaining a sum of 2025 is the same as the probability of obtaining a sum of S. As the number n varies, what is the smallest possible value of S?
[Answer: 341]

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11. What is the only real number $x > 1$ that satisfies the equation below?

$$(\log_3 x)(\log_5 x)(\log_7 x) = (\log_3 x)(\log_5 x) + (\log_5 x)(\log_7 x) + (\log_3 x)(\log_7 x)$$

[Answer: 105]

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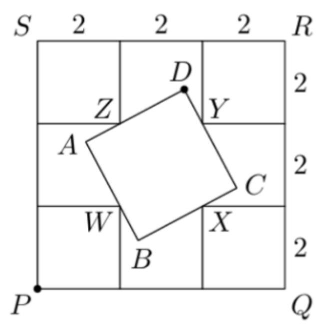
12. In the corners of a square $PQRS$ with side length 6, four smaller squares are placed with side lengths 2, as shown in the diagram below. We will label the points W, X, Y and Z as shown in the diagram. A square $ABCD$ is constructed in such a way that the points W, X, Y, and Z lie on sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively. Find the maximum possible value of $(PD)^2$.



[Answer: 36]

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13. Given that x and y are real numbers and $x^2 + y^2 = 14x + 6y + 6$, what is the largest possible value of $3x + 4y$?
[Answer: 73]

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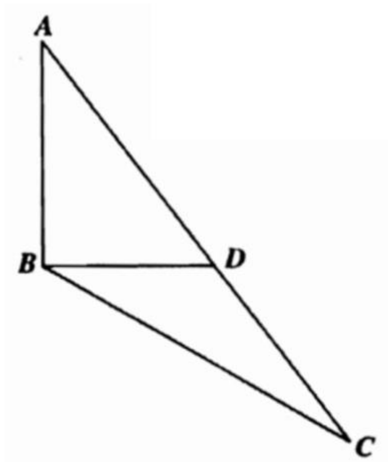
14. In the diagram below, $AB = 4$, $BD = 3$, $AD = 5$, $m\angle DBC = 30°$, and $\overline{ADC}$ is a straight line. $DC = \frac{a\sqrt{b}+c}{d}$, where a, b, c, and d are positive integers, b is not divisible by the square of any prime number, and $\gcd(a, c, d) = 1$. Find $a + b + c + d$.



[Answer: 51]

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15. How many nonempty subsets of {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} have the property that no two elements of the subset differ by more than 5 ? For example, count the subsets {3}, {2, 5, 7} and {5, 6, 7, 8, 9}, but not the subset {1, 3, 5, 7}.
[Answer: 255]

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16. Suppose that $2\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{2}$. Then $x^2 = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
[Answer: 6]

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17. The base-2 representation of the number N is

1 0 1 1 0 1 0 1 0 1 0 1 0 1 $\underline{A}$ $\underline{B}$ $\underline{C}$ 1 1 0

where each of the digits A, B, C is a 0 or a 1. If N is divisible by 7, what are the last seven digits of the base-2 representation of N ? (The answer is a seven-digit number consisting of 1s and 0s, only.)
[Answer: 1010110]

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18. In triangle ABC, $AB = AC$ and we let ω be the unique circle inscribed in the triangle. Suppose that the orthocenter of triangle ABC lies on ω. Then $\cos\angle BAC = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
[Answer: 10]

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19. In the sequence $a_1, a_2, a_3, \ldots$, let $a_k = (k^2 + 1)k!$ and $b_k = a_1 + a_2 + \cdots + a_k$. Then

$$\frac{a_{100}}{b_{100}} = \frac{m}{n}$$

where m and n are relatively prime positive integers. Find $n - m$.
[Answer: 99]

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20. Five people take turns rolling a fair six-sided die numbered from 1 to 6, with each person rolling exactly once. The probability that each person's roll is greater than or equal to the previous person's roll is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.
[Answer: 223]

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21. We will call an n-digit number *sweet* if its n digits are an arrangement of the set $\{1, 2, \ldots, n\}$ and, for $k = 1, 2, \ldots n$, its first k digits form an integer that is divisible by k. For example, 321 is a sweet 3-digit integer because 1 divides 3, 2 divides 32 and 3 divides 321. How many sweet 6-digit numbers are there?
[Answer: 2]

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22. A farmer has 5 cows, 4 pigs, and 7 horses. The farmer will sort the animals into pairs in such a way that no animal is paired with an animal of the same species. Assuming that all the animals are distinguishable, in how many ways can this be done?
[Answer: 100800]

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23. The value of x that minimizes $\sqrt{x^2 + 49} + \sqrt{(8 - x)^2 + 25}$ is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
[Answer: 17]

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24. The longer of the two side lengths of rectangle $AXCY$ is 11, and rectangle $AXCY$ shares diagonal $\overline{AC}$ with square $ABCD$. Assume that B and X lie on the same side of $\overline{AC}$ such that triangle BXC and square $ABCD$ are non-overlapping. The maximum area of triangle BXC across all such configurations is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
[Answer: 137]

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25. For any positive integer n, let $s(n)$ denote the sum of the digits of n. Find the largest positive integer n such that $n = \left(s(n)\right)^2 + 2s(n) - 2$.
[Answer: 397]
