Name: ___________________________________

Grade:___________________________________

School: _________________________________

CONNECTICUT ARML QUALIFICATION TEST
Thursday, March 6, 2025

Pre-test instructions:

- When you receive the link to this test, print out the test and staple it. Do not read it.
- Then gather all you will need for the test: pencils, scrap paper. (Graph paper and protractors are not allowed.)
- Put your phone, all calculators, and any other electronic devices apart from your computer on a surface way out of reach. (You will need your phone at the end of the test in order to take images of your scratchwork.)
- Go to the bathroom now, so that you won't need to during the test.
- Your computer should still be signed on to the Zoom meeting you have been given for this test. Position your computer camera on your desk about 20 inches to the side of the place you will be working. Your entire work area should be visible to the proctors, and it must **not** be possible for other participants to read what you are writing.
- Read the directions below and wait to be told to start the test.

Test directions:

- You will be given 1 hour 45 minutes in which to answer 25 questions.
- All questions carry equal weight.
- All answers are positive integers.
- Write your work in the space provided. You will be required to submit your work at the end of the test in order to confirm the authenticity of your answers. This scratchwork will not be graded.
- All the usual rules for testing apply to this test, including the fact that no communication of any sort with any person is allowed, except with a proctor of this test. (You will be asked to sign a pledge at the end of the test.)
- Books, class notes, etc. are **not** allowed.
- Calculators and the Internet are **not** allowed.

1. There are 10 delegates at a conference: 6 from Party A and 4 from Party B. A committee of 6 delegates is to be formed such that, on the committee, the number of delegates from Party A is greater than the number of delegates from Party B. In how many ways can this be done?

Answer: ______________________

Please note: Only the answers you submit online at the end of the test will count.

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2. Find the largest positive integer n such that $30!$ is divisible by 2^n.

Answer: ______________________

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3. Let α, β, and γ be the solutions of the equation $2x^3 + 3x^2 - 5x - 4 = 0$. Then $\alpha^2 + \beta^2 + \gamma^2 = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Answer: ______________________

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4. Four circles of radius 4 are internally tangent to a circle of radius r, with each of the smaller circles externally tangent to two others. Then $r = a + b\sqrt{c}$, where a, b, and c are positive integers and c is not divisible by the square of any prime number. Find $a + b + c$.

Answer: ______________________

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5. Find the number of values of x with $0 < x < 2\pi$ such that

$$(4^{\cos 2x})^{\sin 2x} = 2$$

Answer: ______________________

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6. Let A, B, C, and D be positive integers (not necessarily distinct) such that $A^2 + B^2 = 20$ and $C^2 - D^2 = 24$. Find the greatest possible value for the sum $A + B + C + D$.

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Answer: ______________________

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7. Let f be a function with the property that, for any real x, $xf(x) + f(x+2) = x^2$. Find $f(8)$.

Answer: ______________________

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8. In the complex plane, let z and w be numbers satisfying $z^6 = 1$ and $w^4 = -1$. Given that 0, z, w, and $z + w$ form a quadrilateral with nonzero area, the minimum possible area of the quadrilateral can be expressed as $\frac{\sqrt{a}-\sqrt{b}}{c}$, where a, b, and c are positive integers, and neither a nor b is divisible by the square of any prime number. Find $a + b + c$.

Answer: ______________________

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9. Point D lies inside triangle ABC so that $AD = 8$, $BD = 5$, and $m\angle ADC = m\angle ADB = m\angle CDB = 120°$.
Given that $\angle ABC$ is a right angle, find the length CD.

Answer: ______________________

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10. Let n be a positive integer. When n dice are rolled, the nonzero probability of obtaining a sum of 2025 is the same as the probability of obtaining a sum of S. As the number n varies, what is the smallest possible value of S ?

Answer: ______________________

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11. What is the only real number $x > 1$ that satisfies the equation below?

$$(\log_3 x)(\log_5 x)(\log_7 x) = (\log_3 x)(\log_5 x) + (\log_5 x)(\log_7 x) + (\log_3 x)(\log_7 x)$$

Answer: ______________________

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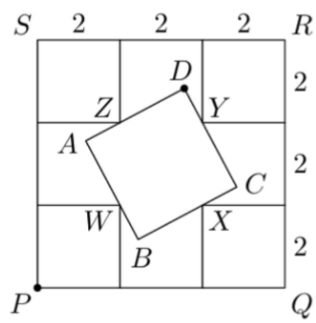
12. In the corners of a square *PQRS* with side length 6, four smaller squares are placed with side lengths 2, as shown in the diagram below. We will label the points W, X, Y and Z as shown in the diagram. A square *ABCD* is constructed in such a way that the points W, X, Y, and Z lie on sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively. Find the maximum possible value of $(PD)^2$.



Answer: ______________________

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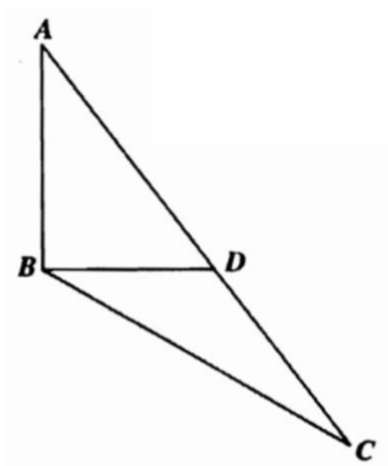
13. Given that x and y are real numbers and $x^2 + y^2 = 14x + 6y + 6$, what is the largest possible value of $3x + 4y$?

Answer: ______________________

14. In the diagram below, $AB = 4$, $BD = 3$, $AD = 5$, $m\angle DBC = 30°$, and $\overline{ADC}$ is a straight line. $DC = \frac{a\sqrt{b}+c}{d}$, where a, b, c, and d are positive integers, b is not divisible by the square of any prime number, and $\gcd(a, c, d) = 1$. Find $a + b + c + d$.



Answer: ______________________

15. How many nonempty subsets of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ have the property that no two elements of the subset differ by more than 5 ? For example, count the subsets {3}, {2, 5, 7} and {5, 6, 7, 8, 9}, but not the subset {1, 3, 5, 7}.

Answer: ______________________

16. Suppose that $2\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{2}$. Then $x^2 = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Answer: ______________________

17. The base-2 representation of the number N is

$$1\,0\,1\,1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,\underline{A}\,\underline{B}\,\underline{C}\,1\,1\,0$$

where each of the digits A, B, C is a 0 or a 1. If N is divisible by 7, what are the last seven digits of the base-2 representation of N? (The answer is a seven-digit number consisting of 1s and 0s, only.)

Answer: ______________________

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18. In triangle ABC, $AB = AC$ and we let ω be the unique circle inscribed in the triangle. Suppose that the orthocenter of triangle ABC lies on ω. Then $\cos \angle BAC = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Answer: ______________________

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19. In the sequence $a_1, a_2, a_3, \dots$, let $a_k = (k^2 + 1)k!$ and $b_k = a_1 + a_2 + \cdots + a_k$. Then

$$\frac{a_{100}}{b_{100}} = \frac{m}{n}$$

where m and n are relatively prime positive integers. Find $n - m$.

Answer: ______________________

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20. Five people take turns rolling a fair six-sided die numbered from 1 to 6, with each person rolling exactly once. The probability that each person's roll is greater than or equal to the previous person's roll is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

Answer: ______________________

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21. We will call an n-digit number *sweet* if its n digits are an arrangement of the set $\{1, 2, \ldots, n\}$ and, for $k = 1, 2, \ldots n$, its first k digits form an integer that is divisible by k. For example, 321 is a sweet 3-digit integer because 1 divides 3, 2 divides 32 and 3 divides 321. How many sweet 6-digit numbers are there?

Answer: ______________________

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22. A farmer has 5 cows, 4 pigs, and 7 horses. The farmer will sort the animals into pairs in such a way that no animal is paired with an animal of the same species. Assuming that all the animals are distinguishable, in how many ways can this be done?

Answer: ______________________

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23. The value of x that minimizes $\sqrt{x^2+49}+\sqrt{(8-x)^2+25}$ is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Answer: ______________________

24. The longer of the two side lengths of rectangle $AXCY$ is 11, and rectangle $AXCY$ shares diagonal $\overline{AC}$ with square $ABCD$. Assume that B and X lie on the same side of $\overline{AC}$ such that triangle BXC and square $ABCD$ are non-overlapping. The maximum area of triangle BXC across all such configurations is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Answer: ______________________

25. For any positive integer n, let $s(n)$ denote the sum of the digits of n. Find the largest positive integer n such that $n = \left(s(n)\right)^2 + 2s(n) - 2$.

Answer: ______________________

END OF TEST

Post-test directions. You have 10 minutes to complete steps 1–3 below.

1. Enter your answers on the Google Form provided in the Zoom chat. Check that you have typed your answers correctly, and then submit the form. (You have 3½ minutes to do this.)

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2. Please complete the following pledge by crossing out the words that do not apply, and add your signature.

   I did / did not abide by exam rules exactly as is expected in a classroom.
   I did / did not complete this test without the help of any person or source.
   I did / did not complete this test without the use of a calculator and/or the Internet.

   Signed:______________________________

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3. Get your phone, and using the Adobe Scan app, create a PDF consisting of all the pages of this test, including the front page (which includes your name) and this page (which includes your signed pledge). For this document, select share, email, and send the link by email to CTARMLTeam@gmail.com. Please include your name in the subject line.

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4. Please remain on the Zoom meeting until you are told that you may leave.
