1.______________________

2.______________________

3.______________________

2025 NEAML

Round 1. -Arithmetic and Number Theory

1. Compute the positive value of the continued fraction $3 + \cfrac{2}{3 + \cfrac{2}{3 + \cfrac{2}{3 + \frac{2}{\cdots}}}}$ in the form $\dfrac{A + \sqrt{B}}{C}$ where A, B, and C are whole numbers. *Note: The pattern continues, with 3's and 2's alternating.*

2. Compute the <u>least</u> value of n for which $\dfrac{n!(n+1)!}{792^3}$ is an integer perfect square.

3. Let $n = 3^{20}7^{25}$. Compute the number of positive integer divisors of n^2 that are less than n but do not divide n.

1. ______________________

2. ______________________

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Round 2 - Algebra 1

1. Given two numbers M and N such that $M + N = 6$ and $\frac{1}{M} + \frac{1}{N} = \frac{6}{7}$, compute $M^2N + MN^2$.

2. In an arithmetic sequence, the first term is 15 and the fifth term is 30. Compute the thirteenth term.

3. Tim uses two hoses to fill a pool. The blue hose could fill the pool in four hours by itself. The green hose could fill the pool in three hours by itself. Compute the integer nearest the number of minutes it would take these hoses working together to fill the pool.

1.____________________________

2.____________________________

3.____________________________

Round 3 - Geometry

1. In a circle centered at O, chord $\overline{PQ}$ is perpendicular to diameter $\overline{RS}$ at X. Given that $PQ = 8$ and $RX = 3$, compute the radius of the circle.

2. Equilateral triangle DUO is inscribed in a circle, and S is a point on minor arc UO. Given that the length of chord $\overline{US}$ is 5 and the length of chord $\overline{SO}$ is 7, compute DS.

3. Two circles, centered at O_1 and O_2, each have radius 4, and each passes through the center of the other. Compute the total area enclosed by the circles.

1. ______________________

2. ______________________

3. ______________________

Round 4 - Algebra 2

1. Compute the value of n for which $n! - (n-1)! = 35280$.

2. The radical expression $\sqrt{18+\sqrt{35}}$ may be expressed in the form $\frac{x+\sqrt{y}}{\sqrt{z}}$ for positive integers x, y, and z. Compute the ordered triple (x, y, z) for which $x+y+z$ is a minimum.

3. Let the zeroes of $f(x) = 2x^3 - 3x - 1$ be p, q, and r. Compute $\frac{1}{p^2} + \frac{1}{q^2} + \frac{1}{r^2}$.

1.______________________________

2.______________________________

3.______________________________

Round 5 - Analytic Geometry

1. The circle with equation $x^2+y^2+2x-6y=12$ has center (h,k) and radius r. Compute $h+k+r^2$.

2. Rhombus $RHOM$ has vertices at $R(5,5)$, $H(4,1)$, $O(0,0)$, and $M(1,4)$. Compute the area of $RHOM$.

3. A circle centered at a point on the y-axis above $(0,4)$ is tangent to the lines $y=x$, $y=-x$, $y=4$, and $x=k$ where $k>4$. Compute k.

1.____________________

2.____________________

3.____________________

Round 6 - Trig and Complex Numbers

1. Given that $i = \sqrt{-1}$, compute $\dfrac{625}{(3i-4)^2}$ in simplest $a + bi$ form.

2. Equilateral triangle ABC has perimeter 12. Quadrilaterals $ACDE$, $BAFG$, and $CBHI$ are squares constructed on the exterior of $\triangle ABC$. Compute the area of hexagon $DEFGHI$ in the form $A + B\sqrt{C}$.

3. The three cube roots of $-4\sqrt{2}+4\sqrt{2}i$ are $A\operatorname{cis}\theta_1$, $A\operatorname{cis}\theta_2$, $A\operatorname{cis}\theta_3$ where $0 \le \theta_1 \le \theta_2 \le \theta_3 \le 2\pi$. Compute the ordered pair (A, θ_2).

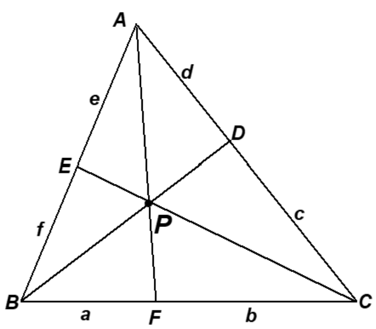
NEAML TEAM ROUND

1.________________ 2.________________ 3.________________

4.________________ 5.________________ 6.________________

1. Compute the term in the expansion of $\left(2x^3 + \dfrac{1}{x^7}\right)^{10}$ **that contains no** x**'s.**

2. ΔABC **has sides of lengths 13, 14, and 15. A cevian is drawn to a trisection point of the longest side. A second cevian is drawn dividing the shortest side into segments in a** $12:1$ **ratio. A third cevian passes through the point of intersection of the first two cevians, and divides the third side of the triangle into two segments. Compute the shortest possible length of one of the six segments.**



3. The height of water in a reservoir is given by $H(t) = 20 \cdot \cos\left(\dfrac{2\pi}{365}(t-80)\right) + 100$ **where** t **is measured in days after April 1 and** H **is measured in feet. Given that** M **is the maximum height of the water over a calendar year in feet and** P **is the length of time in days between maximum heights, compute** $M + P$**.**

4. For the sequence a_n**, suppose that** $a_1 = 20$**,** $a_2 = 19$**, and** $a_n = a_{n-1} + a_{n-2}$ **for** $n \geq 3$**. Of the first 2019 terms, compute the number of terms that are divisible by 3.**

5. Three positive integers a**,** b**, and** c**, with** $a < b < c$**, satisfy** $a + b + c = 40$**. The positive pairwise differences of these numbers are** 5**,** 9**, and** 14**. Compute** $a^2 + c^2$**.**

6. Consider the sequence $2, a, b, 8$**. The first three terms are in a harmonic sequence (where the reciprocal of the middle term is equal to the average of the reciprocals of the other two terms). The last three terms are in an increasing arithmetic sequence. Given that** b **is** $A + B\sqrt{C}$ **when written in simplest form, compute** $A^2 + B^2 + C^2$

2025 NEAML

ANSWERS

Round 1

1. $\frac{3+\sqrt{17}}{2}$
2. 21
3. 500

Round 4.

1. 8
2. $(1, 35, 2)$
3. 9

Round 2.

1. 42
2. 60
3. 103

Round 5.

1. 24
2. 15
3. $4\sqrt{2}+4$

Round 3.

1. $\frac{25}{6}$
2. 12
3. $\frac{64\pi}{3}+8\sqrt{3}$

Round 6.

1. $7+24i$
2. $48+16\sqrt{3}$
3. $\left(2, \frac{11\pi}{12}\right)$

TEAM ROUND

1. 15360
2. $\frac{14}{25}$
3. 485
4. 505
5. 490
6. 17