

Name: \_\_\_\_\_

Grade: \_\_\_\_\_

School: \_\_\_\_\_

## CONNECTICUT ARML QUALIFICATION TEST

Thursday, March 5, 2026

### Pre-test instructions:

- When you receive the link to this test, print out the test and staple it. Do not read it.
- Then gather all you will need for the test: pencils, scrap paper. (Graph paper and protractors are not allowed.)
- Put your phone, all calculators, and any other electronic devices apart from your computer on a surface way out of reach. (You will need your phone at the end of the test in order to take images of your scratchwork.)
- Go to the bathroom now, so that you won't need to during the test.
- Your computer should still be signed on to the Zoom meeting you have been given for this test. Position your computer camera on your desk about 20 inches to the side of the place you will be working. Your entire work area should be visible to the proctors, and it must **not** be possible for other participants to read what you are writing.
- Read the directions below and wait to be told to start the test.

### Test directions:

- You will be given 1 hour 45 minutes in which to answer 25 questions.
- All questions carry equal weight.
- All answers are positive integers.
- Write your work in the space provided. You will be required to submit your work at the end of the test in order to confirm the authenticity of your answers. This scratchwork will not be graded.
- All the usual rules for testing apply to this test, including the fact that no communication of any sort with any person is allowed, except with a proctor of this test. (You will be asked to sign a pledge at the end of the test.)
- Books, class notes, etc. are **not** allowed.
- Calculators and the Internet are **not** allowed.
- No use of any form of AI is allowed during the test. The proctors will be watching carefully to make sure that this rule and all other rules are adhered to.

1. What is the last digit of  $1! + 2! + 3! + \cdots + 2026!$  ?  
(Please note: Only the answers you submit online will count. The answer spaces provided in this question packet are just there to help you organize your work.)

Answer: \_\_\_\_\_

2. The four-digit numbers formed by the digits 1, 2, 3, 4 without repetition are listed in increasing order. What is the 20th number in this list?

Answer: \_\_\_\_\_

3. The point  $(0, 0)$  lies on a circle with center  $(1, 1)$ . The area of the portion of the interior of the circle that lies above the  $x$ -axis is  $\frac{a}{b}\pi + c$ , where  $a$ ,  $b$ , and  $c$  are positive integers and  $a$  and  $b$  are relatively prime. Find  $a + b + c$ .

Answer: \_\_\_\_\_

4. A school's math team consists of 10 students. At least three of those students will be chosen to bring snacks to the next practice. In how many ways can this choice of students be made?

Answer: \_\_\_\_\_

5. If  $x + \frac{1}{x} = 3$ , what is the value of  $x^5 + \frac{1}{x^5}$ ?

Answer: \_\_\_\_\_

6. Three fair six-sided dice (each numbered 1 through 6) will be rolled. The probability that the sum of the numbers resulting is a prime number greater than or equal to 13 is  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .

Answer: \_\_\_\_\_

7. The function  $f$  is defined for all real numbers  $x$  and is given by

$$f(x) = \sin(2026x) + |x - 20| + |x - 26|.$$

What is the minimum value of the function?

Answer: \_\_\_\_\_

8. Suppose that  $Q(x)$  is a quadratic polynomial with a repeated root, and that  $Q(12) = Q(16)$  and  $Q(20) = 24$ . Then  $Q(28) = \frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .

Answer: \_\_\_\_\_

9. Let  $A$  be the set of all integers  $n$  with  $0 \leq n \leq 19$ . How many three-element subsets of  $A$  contain at least two consecutive integers?

Answer: \_\_\_\_\_

10. Let

$$f(x) = x^4 + ax^3 + bx^2 + cx + d$$

be a polynomial. Suppose that  $f(n) = 4n$  for  $n = 1, 2, 3, 4$ . Find  $f(-1)$ .

Answer: \_\_\_\_\_

11. A frog starts at vertex  $A$  of regular hexagon  $ABCDEF$ . On each jump, the frog moves to one of the two adjacent vertices, each with probability  $\frac{1}{2}$ , independently of all previous jumps. The probability that after exactly six jumps the frog is back at  $A$  is  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .

Answer: \_\_\_\_\_

12. Suppose that  $a > 0$ ,  $b > 0$  and  $\log_4 a = \log_6 b = \log_9(a + b)$ . Then

$$\frac{a}{b} = \frac{\sqrt{m} - n}{k},$$

where  $m$ ,  $n$  and  $k$  are positive integers and  $m$  is not divisible by the square of any prime number. Find  $m + n + k$ .

Answer: \_\_\_\_\_

13. Let  $m$  and  $n$  be positive integers greater than or equal to 3 with the property that a convex  $m$ -gon has 19 more diagonals than a convex  $n$ -gon. Find the sum of all possible values of  $m$ .

Answer: \_\_\_\_\_

14. Let  $f$  be a real function such that for all  $x \neq 0, 1$ ,

$$f(x) + f\left(-\frac{1}{x-1}\right) = \frac{9}{4x^2} + f\left(1 - \frac{1}{x}\right).$$

Then  $f\left(\frac{1}{2}\right) = \frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .

Answer: \_\_\_\_\_

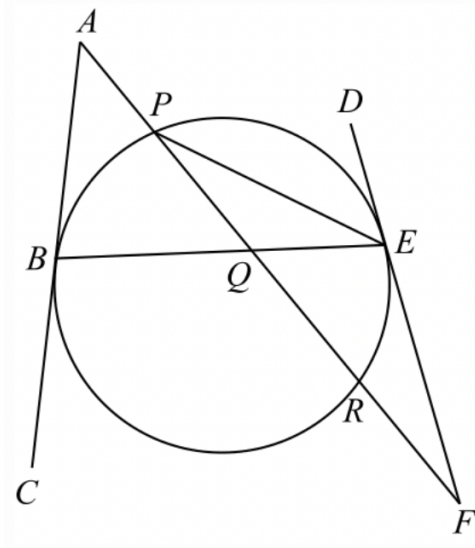
15. In triangle  $ABC$ ,  $AB = 4$ ,  $BC = 3$ , and  $AC = 2$ . The volume of the solid formed by rotating the triangle  $360^\circ$  about the line  $\overline{AC}$  is  $\frac{a\pi}{b}$ , where  $a$  and  $b$  are relatively prime positive integers. Find  $a + b$ .

Answer: \_\_\_\_\_

16. Find the sum of all positive integers  $n$  such that  $\frac{(n+1)^2}{n+23}$  is an integer.

Answer: \_\_\_\_\_

17. In the diagram, line segments  $\overline{AC}$  and  $\overline{DF}$  are tangent to the circle at  $B$  and  $E$ , respectively. Also,  $\overline{AF}$  intersects the circle at  $P$  and  $R$ , and intersects  $\overline{BE}$  at  $Q$ , as shown. If  $m\angle CAF = 35^\circ$ ,  $m\angle DFA = 30^\circ$ , and  $m\angle FPE = 25^\circ$ , then the measure of  $\angle PEQ$  in degrees is  $\frac{p}{q}$ , where  $p$  and  $q$  are relatively prime positive integers. Find  $p + q$ .



Answer: \_\_\_\_\_

18. Suppose that, in triangle  $ABC$ ,  $BC = 4$ ,  $CA = 5$ ,  $AB = 6$ . Then,

$$\sin^6\left(\frac{A}{2}\right) + \cos^6\left(\frac{A}{2}\right) = \frac{m}{n}$$

where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .

Answer: \_\_\_\_\_

19. Given that

$$\begin{aligned}x + y + z &= 10, \\x^2 + y^2 + z^2 &= 42, \\x^4 + y^4 + z^4 &= 1122,\end{aligned}$$

find  $xyz$ .

Answer: \_\_\_\_\_

20. We will say that a rational number  $r = \frac{p}{q}$  is “good” if  $p$  and  $q$  are relatively prime positive integers,  $p < q$ , and the product  $pq$  is a factor of 3600. What is the total number of good rational numbers?

Answer: \_\_\_\_\_

21. The value of  $\sin\left(\frac{\pi}{10}\right)$  is  $\frac{\sqrt{m-n}}{k}$ , where  $m$ ,  $n$ , and  $k$  are positive integers and  $m$  is not divisible by the square of any prime number. Find  $m + n + k$ .

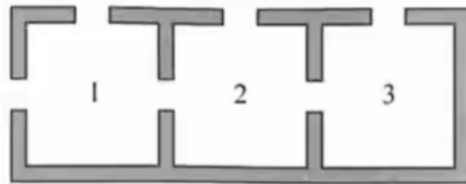
Answer: \_\_\_\_\_

**22.** What is the smallest positive integer  $n$  such that  $n^2 + 11n + 121$  is a perfect square?

Answer: \_\_\_\_\_

23. A box used in a particle experiment is divided into three rooms labeled 1, 2, and 3, arranged in a row, as shown below. If a particle exits the box, it is captured and the experiment ends. Each room has several doors through which the particle may leave the room: Rooms 1 and 2 each have three doors, and Room 3 has two doors. When the particle is in any given room, it is equally likely to leave the room through any of the doors of that room.

Initially the particle is placed in Room 1. The probability that the particle eventually exits the box from Room 1 is  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .



Answer: \_\_\_\_\_

24. Suppose that the complex numbers  $z_1$  and  $z_2$  satisfy the inequalities  $|z_1 + 2z_2| \leq 1$  and  $|z_1^2 + z_1z_2 + z_2^2| \leq 1$ . The largest possible value of  $\max\{|z_1|, |z_2|\}$  is  $\frac{\sqrt{m}}{n}$ , where  $m$  and  $n$  are positive integers and  $m$  is not divisible by the square of any prime number. Find  $m + n$ .

Answer: \_\_\_\_\_

25. The side length of equilateral triangle  $ABC$  is 2, and point  $P$  lies on the circle with center  $A$  and radius 1. The maximum possible value of  $\frac{PB}{PC}$  is  $\frac{\sqrt{m+n}}{k}$ , where  $m$ ,  $n$ , and  $k$  are positive integers and  $m$  is not divisible by the square of any prime number. Find  $m + n + k$ .

Answer: \_\_\_\_\_

Post-test directions. You have 10 minutes to complete steps 1–3 below.

1. Enter your answers on the Google Form provided in the Zoom chat. Check that you have typed your answers correctly, and then submit the form. (You have 3½ minutes to do this.)
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2. Please complete the following pledge by crossing out the words that do not apply, and add your signature.

I did / did not abide by exam rules exactly as is expected in a classroom.

I did / did not complete this test without the help of any person or source.

I did / did not complete this test without the use of a calculator, the Internet, or any form of AI.

Signed: \_\_\_\_\_

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3. Get your phone, and using the Adobe Scan app, create a PDF consisting of all the pages of this test, including the front page (which includes your name) and this page (which includes your signed pledge). For this document, select share, email, and send the link by email to [CTARMLTeam@gmail.com](mailto:CTARMLTeam@gmail.com). Please include your name in the subject line.
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4. Please remain on the Zoom meeting until you are told that you may leave.
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