CT ARML Qualification Test, 2026

1. What is the last digit of $1! + 2! + 3! + \cdots + 2026!$?
[Answer: 3]

2. The four-digit numbers formed by the digits $1, 2, 3, 4$ without repetition are listed in increasing order. What is the 20th number in this list?
[Answer: 4132]

3. The point $(0, 0)$ lies on a circle with center $(1, 1)$. The area of the portion of the interior of the circle that lies above the x-axis is $\frac{a}{b}\pi + c$, where a, b, and c are positive integers and a and b are relatively prime. Find $a + b + c$.
[Answer: 6]

4. A school's math team consists of 10 students. At least three of those students will be chosen to bring snacks to the next practice. In how many ways can this choice of students be made?
[Answer: 968]

5. If $x + \frac{1}{x} = 3$, what is the value of $x^5 + \frac{1}{x^5}$?
[Answer: 123]

6. Three fair six-sided dice (each numbered 1 through 6) will be rolled. The probability that the sum of the numbers resulting is a prime number greater than or equal to 13 is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
[Answer: 10]

7. The function f is defined for all real numbers x and is given by

$$f(x) = \sin(2026x) + |x - 20| + |x - 26|.$$

What is the minimum value of the function?
[Answer: 5]

8. Suppose that $Q(x)$ is a quadratic polynomial with a repeated root, and that $Q(12) = Q(16)$ and $Q(20) = 24$. Then $Q(28) = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
[Answer: 395]

9. Let A be the set of all integers n with $0 \le n \le 19$. How many three-element subsets of A contain at least two consecutive integers?
[Answer: 324]

10. Let

$$f(x) = x^4 + ax^3 + bx^2 + cx + d$$

be a polynomial. Suppose that $f(n) = 4n$ for $n = 1, 2, 3, 4$. Find $f(-1)$.
[Answer: 116]

11. A frog starts at vertex A of regular hexagon $ABCDEF$. On each jump, the frog moves to one of the two adjacent vertices, each with probability $\frac{1}{2}$, independently of all previous jumps. The probability that after exactly six jumps the frog is back at A is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
[Answer: 43]

12. Suppose that $a > 0$, $b > 0$ and $\log_4 a = \log_6 b = \log_9(a + b)$. Then

$$\frac{a}{b} = \frac{\sqrt{m} - n}{k},$$

where m, n and k are positive integers and m is not divisible by the square of any prime number. Find $m + n + k$.
[Answer: 8]

13. Let m and n be positive integers greater than or equal to 3 with the property that a convex m-gon has 19 more diagonals than a convex n-gon. Find the sum of all possible values of m.
[Answer: 33]

14. Let f be a real function such that for all $x \neq 0, 1$,

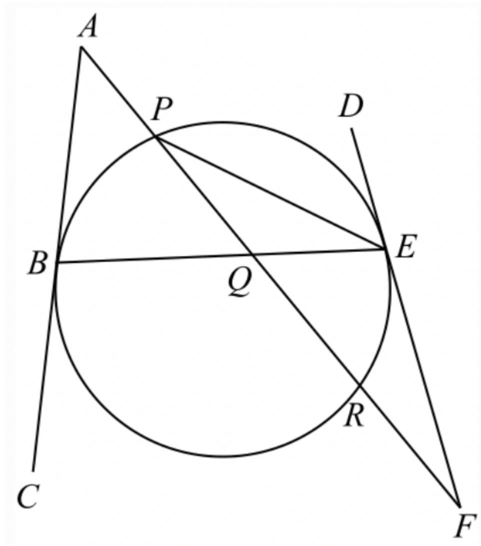
$$f(x) + f\left(-\frac{1}{x-1}\right) = \frac{9}{4x^2} + f\left(1 - \frac{1}{x}\right).$$

Then $f\left(\frac{1}{2}\right) = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
[Answer: 53]

15. In triangle ABC, $AB = 4$, $BC = 3$, and $AC = 2$. The volume of the solid formed by rotating the triangle 360° about the line $\overline{AC}$ is $\frac{a\pi}{b}$, where a and b are relatively prime positive integers. Find $a + b$.
[Answer: 53]

16. Find the sum of all positive integers n such that $\frac{(n+1)^2}{n+23}$ is an integer.
[Answer: 799]

17. In the diagram, line segments $\overline{AC}$ and $\overline{DF}$ are tangent to the circle at B and E, respectively. Also, $\overline{AF}$ intersects the circle at P and R, and intersects $\overline{BE}$ at Q, as shown. If $m\angle CAF = 35°$, $m\angle DFA = 30°$, and $m\angle FPE = 25°$, then the measure of $\angle PEQ$ in degrees is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.



[Answer: 67]

18. Suppose that, in triangle ABC, $BC = 4$, $CA = 5$, $AB = 6$. Then,

$$\sin^6\left(\frac{A}{2}\right) + \cos^6\left(\frac{A}{2}\right) = \frac{m}{n}$$

where m and n are relatively prime positive integers. Find $m + n$.
[Answer: 107]

19. Given that

$$x + y + z = 10,$$
$$x^2 + y^2 + z^2 = 42,$$
$$x^4 + y^4 + z^4 = 1122,$$

find xyz.
[Answer: 26]

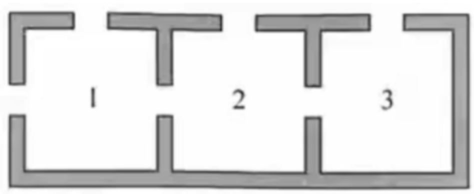
20. We will say that a rational number $r = \frac{p}{q}$ is "good" if p and q are relatively prime positive integers, $p < q$, and the product pq is a factor of 3600. What is the total number of good rational numbers?
[Answer: 112]

21. The value of $\sin\left(\frac{\pi}{10}\right)$ is $\frac{\sqrt{m}-n}{k}$, where m, n, and k are positive integers and m is not divisible by the square of any prime number. Find $m + n + k$.
[Answer: 10]

22. What is the smallest positive integer n such that $n^2 + 11n + 121$ is a perfect square?
[Answer: 24]

23. A box used in a particle experiment is divided into three rooms labeled 1, 2, and 3, arranged in a row, as shown below. If a particle exits the box, it is captured and the experiment ends. Each room has several doors through which the particle may leave the room: Rooms 1 and 2 each have three doors, and Room 3 has two doors. When the particle is in any given room, it is equally likely to leave the room through any of the doors of that room.

Initially the particle is placed in Room 1. The probability that the particle eventually exits the box from Room 1 is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.



[Answer: 23]

24. Suppose that the complex numbers z_1 and z_2 satisfy the inequalities $|z_1 + 2z_2| \leq 1$ and $|z_1^2 + z_1 z_2 + z_2^2| \leq 1$. The largest possible value of $\max\{|z_1|, |z_2|\}$ is $\frac{\sqrt{m}}{n}$, where m and n are positive integers and m is not divisible by the square of any prime number. Find $m + n$.
[Answer: 18]

25. The side length of equilateral triangle ABC is 2, and point P lies on the circle with center A and radius 1. The maximum possible value of $\frac{PB}{PC}$ is $\frac{\sqrt{m}+n}{k}$, where m, n, and k are positive integers and m is not divisible by the square of any prime number. Find $m + n + k$.
[Answer: 18]