CONNECTICUT STATE ASSOCIATION OF MATH LEAGUES
State Math Match, 2026: Solutions

1-1 Twelve kids earn \$278 in 5 days. At the same rate, how many dollars will 15 kids earn in 8 days? (Do not include a unit in your answer.)
[Answer: 556]

Solution

$$\frac{\$278}{12 \text{ kids } x \;\; 5 \text{ days}} = \frac{\$m}{15 \text{ kids } x \;\; 8 \text{ days}} \;--> \frac{15\ x\ 8\ x\ \$278}{12\ x\ 5} = 2\ x\ \$278 = \$556$$

1-2 Suppose that $2.1_4 + n = 71.2_8$, where the subscripts denote the bases of the numeral systems. Find the value of n, expressing your answer in base 2.
[Answer: 110111]

Solution

$2.1_4 = 2.25_{10}$
$71.2_8 = 57.25_{10}$
$57.25_{10} - 2.25_{10} = 55_{10}$
$55_{10} = 110111_2$

1-3 The positive integer N has exactly four positive divisors (including 1 and N), and the sum of these positive divisors is 180. Find the sum of the possible values of N.
[Answer: 263]

Solution.
A positive integer has exactly four positive divisors in exactly two cases:

- $N = p^3$, where p is prime, with divisors $1, p, p^2, p^3$;
- $N = pq$, where p and q are distinct primes, with divisors $1, p, q, pq$.

We are told that the sum of the divisors is 180.

Case 1: $N = p^3$.
The sum of the divisors is

$$1 + p + p^2 + p^3.$$

Set this equal to 180:

$$1 + p + p^2 + p^3 = 180.$$

Testing nearby primes,

$$p = 5 \Rightarrow 1 + 5 + 25 + 125 = 156,$$

$$p = 7 \Rightarrow 1 + 7 + 49 + 343 = 400.$$

Since the sum increases rapidly and no prime gives 180, there are no solutions in this case.

Case 2: $N = pq$ with distinct primes $p < q$.
The sum of the divisors is

$$1 + p + q + pq = (p + 1)(q + 1).$$

Thus

$$(p + 1)(q + 1) = 180.$$

Let $a = p + 1$ and $b = q + 1$. Then $ab = 180$, and $a - 1$ and $b - 1$ must both be prime.
List factor pairs of 180 with $a < b$ and $a \geq 3$:

$$(3, 60),\ (4, 45),\ (5, 36),\ (6, 30),\ (9, 20),\ (10, 18),\ (12, 15).$$

Check which give primes:

$$(3, 60):\ (2, 59) \text{ both prime} \Rightarrow N = 2 \cdot 59 = 118,$$

$$(6, 30):\ (5, 29) \text{ both prime} \Rightarrow N = 5 \cdot 29 = 145.$$

All other pairs fail since one of $a - 1$ or $b - 1$ is not prime.

The possible values of N are 118 and 145, so the required sum is

$$118 + 145 = \boxed{263}.$$

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2-1 If $\sqrt{x+4}-1=\sqrt{x+2}$, then $x = (-1)^n \cdot \frac{a}{b}$, where a and b are relatively prime positive integers and n is 0 or 1. Find $a + b + n$.

[Answer: 12]

Solution

$$\sqrt{x+4}-1 = \sqrt{x+2}$$
$$\sqrt{x+2}+1 = \sqrt{x+4}$$
$$\left(\sqrt{x+2}+1\right)^2 = x+4$$
$$x+2+2\sqrt{x+2}+1 = x+4$$
$$2\sqrt{x+2} = 1$$
$$4(x+2) = 1$$
$$x+2 = \frac{1}{4}$$
$$x = -\frac{7}{4}$$
$$(-1)^n \cdot \frac{a}{b} = -\frac{7}{4}$$
$$n = 1, a = 7, b = 4$$
$$1+7+4 = 12$$

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2-2 If $\frac{3a+4b}{4a-3b} = 2$, then $\frac{4a+3b}{3a-4b} = (-1)^n \cdot \frac{p}{q}$, where p and q are relatively prime positive integers and n is 0 or 1. Find $p + q + n$.

[Answer: 13]

Solution

Start with $\frac{3a+4b}{4a-3b} = 2$ solve for a

$$4a - 3b = 2(4a - 3b)$$
$$3a + 4b = 8a - 6b$$
$$-5a = -10b$$
$$a = 2b$$

Using $\frac{4a+3b}{3a-4b}$

The numerator is: $4(2b) + 3b = 11b$
The numerator is: $3(2b) - 4b = 2b$

$$\frac{4a + 3b}{3a - 4b} = \frac{11}{2}$$
$$\frac{11}{2} = (-1)^n \cdot \frac{p}{q}$$
$$p = 11, q = 2, and\ n = 0$$
$$11 = 2 + 0 = 13$$

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2-3 Define the function $f(x) = \left(x - \frac{1}{2}\right)^3 + \frac{1}{4}$. The value of

$$f\left(\frac{1}{2026}\right)+f\left(\frac{2}{2026}\right)+f\left(\frac{3}{2026}\right)+\ldots+f\left(\frac{2024}{2026}\right)+f\left(\frac{2025}{2026}\right)$$

is $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

[Answer: 2029]

Solution.
Let

$$g(x) = \left(x - \frac{1}{2}\right)^3.$$

Then

$$f(x) = g(x) + \frac{1}{4},$$

so the required sum is

$$\sum_{k=1}^{2025} f\left(\frac{k}{2026}\right) = \sum_{k=1}^{2025} g\left(\frac{k}{2026}\right) + \sum_{k=1}^{2025} \frac{1}{4}.$$

Note that

$$g\left(\frac{k}{2026}\right) = \left(\frac{k}{2026} - \frac{1}{2}\right)^3 \quad \text{and} \quad g\left(\frac{2026-k}{2026}\right) = \left(\frac{2026-k}{2026} - \frac{1}{2}\right)^3 = \left(\frac{1}{2} - \frac{k}{2026}\right)^3 = -g\left(\frac{k}{2026}\right).$$

Thus the terms cancel in pairs:

$$g\left(\frac{1}{2026}\right) = -g\left(\frac{2025}{2026}\right), \quad g\left(\frac{2}{2026}\right) = -g\left(\frac{2024}{2026}\right), \quad \ldots$$

There are 2025 terms, an odd number, so after pairing cancellations the sum equals the middle term:

$$g\left(\frac{1013}{2026}\right) = g\left(\frac{1}{2}\right) = 0.$$

Therefore,

$$\sum_{k=1}^{2025} g\left(\frac{k}{2026}\right) = 0,$$

and the original sum becomes

$$\sum_{k=1}^{2025} f\left(\frac{k}{2026}\right) = \sum_{k=1}^{2025} \frac{1}{4} = \frac{2025}{4}.$$

So $\frac{a}{b} = \frac{2025}{4}$, and $a + b = 2025 + 4 = 2029$.

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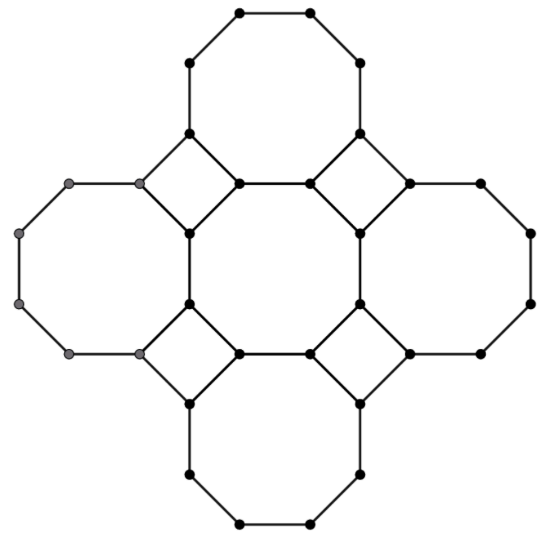
3-1 The points A, B, C, and D are consecutive vertices of a regular polygon. Sides $\overline{AB}$ and $\overline{DC}$ are extended, and intersect at point P. The measure of $\angle BPC$ is 132°. How many sides does the polygon have?
[Answer: 15]

Solution

The extended lines form an Isosceles triangle with base angles of 24 degrees. These are the exterior angles of the regular polygon. 360 divided by the exterior angle give 15 sides

$180 - 132 = 48$
$\frac{48}{2} = 24$
$\frac{360}{24} = 15$

3-2 A tiling pattern consists of squares and regular octagons, as shown in the diagram. If the area of one of the squares is 2 square centimeters, the area of one of the octagons is $a + b\sqrt{c}$, where a, b, and c are positive integers and c is not divisible by the square of any prime number. Find $a + b + c$.
[Answer: 10]



Solution

Forumula for Area of an Octagon is $2\left(1+\sqrt{2}\right)s^2$ where s is the length of a side

$s = \sqrt{2}$

$2\left(1+\sqrt{2}\right)\left(\sqrt{2}\right)^2 = 4 + 4\sqrt{2}$

$a = 4,\ b = 4,\ c = 2$

$4 + 4 + 2 = 10$

CONNECTICUT STATE ASSOCIATION OF MATH LEAGUES
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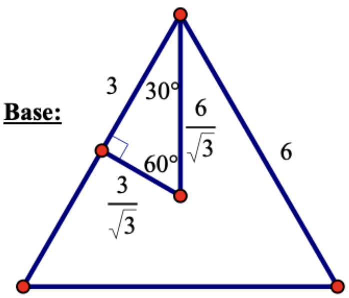
3-3 A regular tetrahedron (a triangular pyramid where all the faces are equilateral triangles) is inscribed in a sphere (meaning that all the vertices of the tetrahedron lie on the sphere). The tetrahedron has edges of length 6. The volume of the sphere is $a\pi\sqrt{b}$, where a and b are positive integers and b is not divisible by the square of any prime number. Find $a + b$.
[Answer: 33]

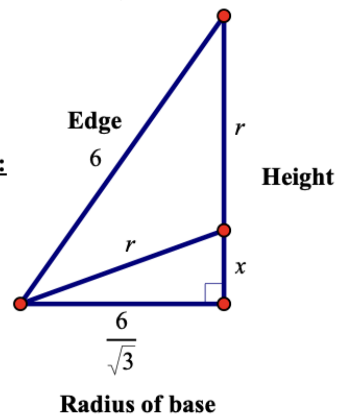
Solution

- To calculate the volume of the sphere, we need to find the radius of the sphere, r.
- Since the base of the tetrahedron is an equilateral triangle, half the edge length, the apothem of the base, and the radius of the base form a 30-60-90 triangle. If the edge is 6, half the edge is 3, making the apothem $\frac{3}{\sqrt{3}}$ and the radius of the base $\frac{6}{\sqrt{3}}$ (see diagram below)

Base:



Cross Section:



- Taking a cross-section of the tetrahedron, the radius of the base, the height of the tetrahedron, and an edge form a right triangle. The radius of the sphere, r, goes from a vertex to the center of the sphere (on the height), and from the center of the sphere to the top vertex of the tetrahedron. See the diagram above.
- Use the Pythagorean theorem to solve for the height of the tetrahedron h:

$$h^2 + \left(\frac{6}{\sqrt{3}}\right)^2 = 6^2 \to h^2 + \frac{36}{3} = 36 \to h^2 + 12 = 36 \to h^2 = 24 \to h = \sqrt{24} = 2\sqrt{6}$$

- The radius is $r = h - x$, so $r = 2\sqrt{6} - x$. Use the Pythagorean theorem on the shaded triangle to solve for x:

$$x^2 + \left(\frac{6}{\sqrt{3}}\right)^2 = r^2 \to x^2 + \left(\frac{6}{\sqrt{3}}\right)^2 = \left(2\sqrt{6} - x\right)^2 \to x^2 + \frac{36}{3} = 24 - 4\sqrt{6}x + x^2 \to 12 = 24 - 4\sqrt{6}x \to$$

$$-12 = -4\sqrt{6}x \to x = \frac{12}{4\sqrt{6}} = \frac{3}{\sqrt{6}} = \frac{\sqrt{6}}{2}$$

- Then the radius of the sphere is: $r = h - x = 2\sqrt{6} - \frac{\sqrt{6}}{2} = \frac{4\sqrt{6} - \sqrt{6}}{2} = \frac{3\sqrt{6}}{2}$
- The volume of the sphere is then:

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{3\sqrt{6}}{2}\right)^3 = \frac{\cancel{4}}{\cancel{3}}\pi\left(\frac{3\cdot 3\cdot 3\cdot \cancel{6}\cdot\sqrt{6}}{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}}\right) = 27\pi\sqrt{6}$$

- So if $a\pi\sqrt{b} = 27\pi\sqrt{6}$, then $a = 27$ and $b = 6$, so $a + b = 27 + 6 = 33$

4-1 The zeros of the polynomial function $f(x) = x^3 + px^2 + qx + r$ are -4, 1, and 2. Find $|p| + |q| + |r|$.
[Answer: 19]

Solution

The zeros of the polynomial function $f(x) = x^3 + px^2 + qx + r$ are −4, 1, and 2.

$$f(x) = (x+4)(x-1)(x-2)$$

$$f(x) = (x^2+3x-4)(x-2)$$

$$f(x) = x^3 + x^2 - 10x + 8$$

Therefore p = 1, q = -10, and r = 8

Find |p| + |q| + |r|. |1| + |-10| + |8|=19.

4-2 For $x > \frac{7}{3}$, the function f is defined by $f(x) = 3x^2 - 14x - 3$. The function g is defined by $g(x) = x^3 + 62$ for all real numbers x. Find $f^{-1}\left(g^{-1}(70)\right)$.
[Answer: 5]

Solution

$\left(g^{-1}(70)\right)$ ➔ $70 = x^3 + 62$
$8 = x^3$ ➔ x = 2

$f^{-1}(2)$ ➔ $2 = 3x^2 - 14x - 3$
$0 = 3x^2 - 14x - 5$

$0 = (3x+1)(x-5)$
$x = 5, or\ x = -\frac{1}{3}$

$-\frac{1}{3}$ *is not greater than* $\frac{7}{3}$

Therefore x = 5

4-3 Let a and b be those real numbers such that $60^a = 3$ and $60^b = 5$. Let $p = \frac{1+a+b}{2(1-b)}$. Find 12^p.
[Answer: 30]

Solution.
We are given

$$60^a = 3 \quad \text{and} \quad 60^b = 5.$$

We also have

$$60 = 12 \cdot 5.$$

First find $1 - b$:

$$60^{1-b} = \frac{60}{60^b} = \frac{60}{5} = 12.$$

Thus

$$12 = 60^{1-b}.$$

Next, compute 60^{1+a+b}:

$$60^{1+a+b} = 60 \cdot 60^a \cdot 60^b = 60 \cdot 3 \cdot 5 = 900.$$

Now recall

$$p = \frac{1+a+b}{2(1-b)}.$$

Then

$$12^p = \left(60^{1-b}\right)^{\frac{1+a+b}{2(1-b)}} = 60^{\frac{1+a+b}{2}} = \sqrt{60^{1+a+b}} = \sqrt{900} = 30.$$

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5-1 The points $A(1,\ 2)$ and $B(7,\ 12)$ are given. Let M be the midpoint of line segment $\overline{AB}$, and let l be the line through M perpendicular to $\overline{AB}$. The x-coordinate of the point where l intersects the x-axis is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.
[Answer: 50]

Solution

Midpoint of AB $=\left(\frac{1+7}{2}, \frac{2+12}{2}\right)$ ➔(4,7)
Slope of AB $= \frac{12-2}{7-1} = \frac{5}{3}$
Line L: $y = -\frac{3}{5}(x-4) + 7$
$y = -\frac{3}{5}x + \frac{47}{5}$
x-intercept of line L: $0 = -\frac{3}{5}x + \frac{47}{5}$

$$-\frac{47}{5} = -\frac{3}{5}x$$

$$x = \frac{47}{3}$$

p = 47, q = 3 Therefore 47+ 3 = 50

5-2 A hyperbola has vertices at $(-6, 0)$ and $(6, 0)$ and a focus at $(7, 0)$. The line $x = 7$ intersects the hyperbola at the points M and N. The length $MN = \frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.
[Answer: 16]

Solution

Equation of Hyperbola: $\frac{x^2}{36} - \frac{y^2}{b^2} = 1$, $a^2 + b^2 = c^2$

$$6^2 + b^2 = 7^2$$

$$b^2 = 13$$

Equation of Hyperbola: $\frac{x^2}{36} - \frac{y^2}{13} = 1$

When x = 7, $\frac{49}{36} - \frac{y^2}{13} = 1$

$$\frac{49}{36} - 1 = \frac{y^2}{13}$$

$$\frac{13}{36} = \frac{y^2}{13}$$

$$\frac{169}{36} = y^2$$

$$\frac{13}{6} = y$$

Double the distance from the x-axis and $\frac{13}{6} = \frac{13}{3}$

Then p = 13, q=3 and p+q = 16

5-3 The point P lies in the same plane as square $ABCD$, where $AB = 1$. Let $AP = u$, $BP = v$, and $CP = w$, and suppose that $u^2 + 2v^2 = 2w^2$. The maximum possible value of the distance DP is $p + \sqrt{q}$, where p and q are positive integers. Find $p + q$.
[Answer: 9]

Solution.
Let

$$A = (0,0), \quad B = (1,0), \quad C = (1,1), \quad D = (0,1),$$

and let $P = (x, y)$.
Then

$$u^2 = AP^2 = x^2 + y^2,$$
$$v^2 = BP^2 = (x-1)^2 + y^2,$$
$$w^2 = CP^2 = (x-1)^2 + (y-1)^2.$$

We are given

$$u^2 + 2v^2 = 2w^2.$$

Substitute the coordinate expressions:

$$x^2 + y^2 + 2\big((x-1)^2 + y^2\big) = 2\big((x-1)^2 + (y-1)^2\big).$$

Expand both sides:

$$x^2 + y^2 + 2(x-1)^2 + 2y^2 = 2(x-1)^2 + 2(y-1)^2.$$

Cancel the common term $2(x-1)^2$ and simplify:

$$x^2 + 3y^2 = 2(y^2 - 2y + 1).$$

Thus

$$x^2 + 3y^2 = 2y^2 - 4y + 2,$$

so

$$x^2 + y^2 + 4y = 2.$$

Complete the square:

$$x^2 + (y^2 + 4y) = 2 \quad \Longrightarrow \quad x^2 + (y+2)^2 = 6.$$

Thus P lies on a circle with center

$$O = (0, -2)$$

and radius

$$r = \sqrt{6}.$$

We want to maximize DP, where $D = (0,1)$. The distance from D to the center is $1 - (-2) = 3$.

For a fixed circle, the maximum distance from an external point to a point on the circle equals the distance from the point to the center plus the radius. Hence

$$DP_{\text{max}} = OD + r = 3 + \sqrt{6}.$$

Thus the maximum value is $3 + \sqrt{6}$, so $p = 3$, $q = 6$, and

$$p + q = \boxed{9}.$$

6-1 Let A and B be acute angles, where $\sin A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$. Then $\tan(A + B) = \frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.
[Answer: 89]

Solution

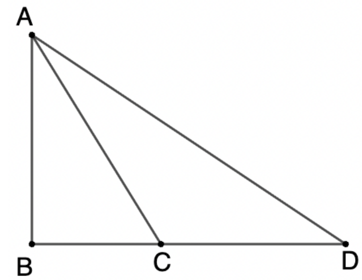
$$\tan(a + b) = \frac{(\tan a + \tan b)}{1 - 3\tan a \cdot \tan b}$$

$$\sin A = \frac{3}{4} \; then \; \tan A = \frac{3}{4} \qquad \sin B = \frac{5}{13} \; then \; \tan B = \frac{5}{12}$$

$$\frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{\frac{14}{12}}{1 - \frac{5}{16}} = \frac{\frac{7}{6}}{\frac{11}{16}} = \frac{7}{6} \cdot \frac{16}{11} = \frac{56}{33}$$

$$56 + 33 = 89$$

6-2 The diagram below shows triangle ABD, with C lying on $\overline{BD}$. If $\overline{AB} \perp \overline{BD}$, $m\angle BCA = 60°$, $m\angle BDA = 30°$, and $DC = 100$, then $AB = p\sqrt{q}$, where p and q are integers and q is not divisible by the square of any prime number. Find $p + q$.



[Answer: 53]

Solution

Given Triangles ACB *is* $30°, 60°, 90°$ *and* DAB *is* $30°, 60°, 90°$

Let $BC = x$ *then* $AB = x\sqrt{3}$, $BC = x + 100$

$$x + 100 = \sqrt{3}\left(x\sqrt{3}\right)$$

$x + 100 = 3x$, $x = 50$, $AB = 50\sqrt{3}$, $50 + 3 = 53$

6-3 In triangle ABC, $BC = 27$, $AB = 48$, and $m\angle C = 3(m\angle A)$. Find the length AC.
[Answer: 35]

Solution.
Let $A = x$. Then $C = 3x$ and

$$B = 180^\circ - (A + C) = 180^\circ - 4x.$$

Let $a = BC = 27$, $b = AC$, and $c = AB = 48$. By the Law of Sines,

$$\frac{a}{\sin A} = \frac{c}{\sin C} \quad \Rightarrow \quad \frac{27}{\sin x} = \frac{48}{\sin 3x}.$$

Thus

$$27 \sin 3x = 48 \sin x.$$

Using $\sin 3x = 3 \sin x - 4 \sin^3 x$,

$$27(3 \sin x - 4 \sin^3 x) = 48 \sin x.$$

Since $\sin x \neq 0$,

$$81 - 108 \sin^2 x = 48 \quad \Rightarrow \quad \sin^2 x = \frac{11}{36}.$$

Hence

$$\sin x = \frac{\sqrt{11}}{6}, \qquad \cos x = \frac{5}{6}.$$

Now

$$\sin 2x = 2 \sin x \cos x = \frac{5\sqrt{11}}{18}, \qquad \cos 2x = \cos^2 x - \sin^2 x = \frac{7}{18}.$$

Thus

$$\sin B = \sin(4x) = 2 \sin 2x \cos 2x = \frac{35\sqrt{11}}{162}.$$

Using the Law of Sines again,

$$b = \frac{a \sin B}{\sin A} = 27 \cdot \frac{\frac{35\sqrt{11}}{162}}{\frac{\sqrt{11}}{6}} = 35.$$

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T-1 How many positive integers less than or equal to 155 leave a remainder of 1 when divided by 3 and a remainder of 2 when divided by 4?
[Answer: 13]

Solution.
Call a number that leaves a remainder of 1 when divided by 3 a $3k+1$ *number*, and a number that leaves a remainder of 2 when divided by 4 a $4k+2$ *number*.
Consider numbers by their remainder when divided by 12.

Step 1: Describe the $3k+1$ numbers. A $3k+1$ number leaves remainder 1 upon division by 3. Among the residues $0, 1, 2, \ldots, 11$, the ones that are 1 more than a multiple of 3 are

$$1,\ 4,\ 7,\ 10.$$

So the $3k+1$ numbers are exactly the numbers of the forms

$$12k+1, \quad 12k+4, \quad 12k+7, \quad 12k+10.$$

Step 2: Describe the $4k+2$ numbers. A $4k+2$ number leaves remainder 2 upon division by 4. Among the residues $0, 1, 2, \ldots, 11$, the ones that are 2 more than a multiple of 4 are

$$2,\ 6,\ 10.$$

So the $4k+2$ numbers are exactly the numbers of the forms

$$12k+2, \quad 12k+6, \quad 12k+10.$$

Step 3: Overlap. To satisfy both conditions, the number must appear in both lists, so it must be of the form

$$12k+10.$$

Step 4: Count them up to 155. We need

$$1 \le 12k+10 \le 155.$$

The smallest occurs at $k=0$: $12(0)+10=10$. The largest k satisfies

$$12k+10 \le 155 \quad \Rightarrow \quad 12k \le 145 \quad \Rightarrow \quad k \le 12.$$

Thus k can be $0, 1, 2, \ldots, 12$, which is 13 values.

T-2 A function f is defined so that if n is an odd integer, then $f(n) = n - 1$, and if n is an even integer, then $f(n) = n^2 - 1$. Determine the sum of the squares of all integers n for which $f(f(n)) = 3$.
[Answer: 10]

Solution

$f(f(n)) = (n-1)^2 - 1 \quad or \quad (n^2 - 1) - 1$

$(n-1)^2 - 1 = 3$, $(n-1)^2 = 4$, $n - 1 = \pm 2$, $n = 3, -1$

$or \quad (n^2 - 1) - 1 = 3$, $n^2 - 1 - 1 = 3$, $n^2 = 5$, *no integer solutions*

$3^2 + (-1)^2 = 9 + 1 = 10$

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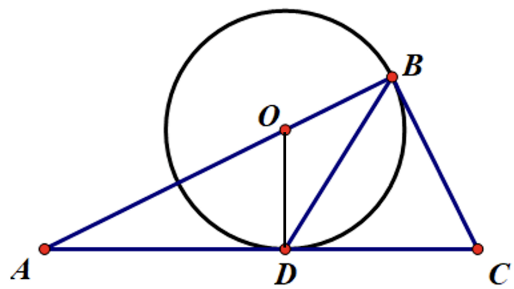
T-3 Circle O, shown in the diagram below, has radius 8. $\overline{AC}$ is tangent to the circle at D, $\overline{BC}$ is tangent to the circle at B, and $AD = 15$. The area of ΔBDC is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.



[Answer: 4051]

Solution

- $\angle ADO$ and $\angle ABC$ are right angles as they are tangents to $\odot O$
- $OD = 8$ as it is a radius, so $AO = 17$ as ΔADO is a Pythagorean triple (8-15-17)
- $AB = AO + OB = 17 + 8 = 25$
- $\Delta ABC \sim \Delta ADO$, so $\frac{AD}{AB} = \frac{OD}{BC} \rightarrow \frac{15}{25} = \frac{8}{BC} \rightarrow BC = \frac{25(8)}{15} = \frac{40}{3}$
- $CD = \frac{40}{3}$ as well, since it is a common tangent to $\odot O$ with BC
- To find the height of ΔBDC, drop an altitude from B to $\overline{DC}$, intersecting at point E. Use similar triangles $\Delta ADO \sim \Delta AEB$ to find BE:
  $\frac{AO}{AB} = \frac{OD}{BE} \rightarrow \frac{17}{25} = \frac{8}{BE} \rightarrow BE = \frac{25(8)}{17} = \frac{200}{17}$
- So the area of $\Delta BDC = \frac{1}{2}bh = \frac{1}{2}(DC)(BE) = \frac{1}{2}\left(\frac{40}{3}\right)\left(\frac{200}{17}\right) = \frac{4000}{51} = \frac{a}{b}$
- So $a = 4000$ and $b = 51$, thus $a + b = 4000 + 51 = 4051$

CONNECTICUT STATE ASSOCIATION OF MATH LEAGUES
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T-4 Evaluate

$$\frac{(1!)(2!)(3!)\cdots(10!)}{(1!)^2(3!)^2(5!)^2(7!)^2(9!)^2}$$

[Answer: 3840]

Solution

$$\frac{1!\cdot 2!\cdot 3!\cdot 4!\cdot 5!\cdot 6!\cdot 7!\cdot 8!\cdot 9!\cdot 10!}{(1!)^2(3!)^2(5!)^2(7!)^2(9!)^2}$$

$$\frac{2!\cdot 4!\cdot 6!\cdot 8!\cdot 10!}{(1!)(3!)(5!)(7!)(9!)}$$

$$\frac{2!}{(1!)}=2, \frac{4!}{(3!)}=4, \frac{6!}{(5!)}=6, \frac{8!}{(7!)}=8, and\ \frac{10!}{(9!)}=10$$

$$(2)(4)(6)(8)(10)=3840$$

T-5 An ellipse has equation $7x^2 + 12y^2 + 28x - 24y - 44 = 0$. The eccentricity of the ellipse is $\frac{\sqrt{p}}{q}$, where p and q are positive integers and p is not divisible by the square of any prime number. Find $p + q$.
[Answer: 21]

Solution

Let's put the ellipse into standard form.
Group and complete the squares:

$$7(x^2+4x)+12(xy^2-2y)=44$$
$$7[(x+2)^2-4]+12[(y-1)^2-1]=44$$
$$7(x+2)^2+12(y-1)^2=44+28+12$$
$$7(x+2)^2+12(y-1)^2=84$$
$$\frac{(x+2)^2}{\frac{84}{7}}+\frac{(y-1)^2}{\frac{84}{12}}=1$$
$$\frac{(x+2)^2}{12}+\frac{(y-1)^2}{7}=1$$
$$a^2=12\quad b^2=7$$

Eccentricity = $\sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{7}{12}} = \sqrt{\frac{5}{12}} = \frac{\sqrt{60}}{12} = \frac{\sqrt{15}}{6}$

Therefore, p = 15 and q = 6 then 15 + 6 = 21

T-6 Find the number of integers k, with $0 < k < 18$, such that

$$\frac{5\sin(10k°)-2}{(10k°)} \geq 2$$

[Answer: 13]

Solution

$Let\ x = \sin 10k°, \quad Solving\ \frac{(5x-2)}{x^2} \geq 2,$

$0 \geq 2x^2 - 5x + 2,\ 0 \geq (2x-1)(x-2)\ gives\ \sin 10k° \geq \frac{1}{2}$

$\sin 30^o = \frac{1}{2},\ \sin 90^o = 1, \sin 150^o = \frac{1}{2},$

$\sin\ is\ continuous\ thus\ k \in \{3,4,5,6,7,8,9,10,11,12,13,14,15\}$

$n = 13\ integers$