2013 CT-ARML runoff **PLEASE PRINT ALL INFO Name** _____
Round 1 (1 hour) **NO CALCULATORS School** _____

1) How many digits are in the base 10 number $4^{22} \times 5^{40}$? For example, 9001 has four digits.

2) The ratio of children to adults at a party is 2:3. A busload of 30 more children arrives at the party, and now the ratio of children to adults is 3:2. How many people were at the party before the bus arrived?

3) Find the area of the region consisting of all points (x, y) satisfying $1 \le |x| + |y| \le 2$.

4) A circle is inscribed in quadrilateral ABCD. If AB = 4, BC = 5, and CD = 8, what is DA?

5) What is the 4-tuple (w, x, y, z) which satisfies all four of the following equations?

$$\begin{cases} w + x + y = 4 \\ x + y + z = -5 \\ y + z + w = 0 \\ z + w + x = -8 \end{cases}$$

6) Circles of radius 10 and 17 intersect at two points. The segment connecting these points of intersection has length 16. List all possible values for the distance between the centers of the circles.

7) What is the value of the continued fraction:

$$3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \ldots}}} ?$$

The 3's and 4's alternate indefinitely.

8) The operation $\otimes$ is defined for all nonzero numbers by $a \otimes b = \frac{a^2}{b}$.

Determine $[(1 \otimes 2) \otimes 3] - [1 \otimes (2 \otimes 3)]$.

9) In triangle ABC, $3\sin A + 4\cos B = 6$ and $4\sin B + 3\cos A = 1$. Find the measure (in degrees) of angle C.

10) Simplify: $\sqrt{\frac{8^{10} + 4^{10}}{8^4 + 4^{11}}}$.

11) Alice, Bill, Chris, Don and Emily take a test. In how many different orders can they finish if ties are allowed? (For example, if there were three people, it would be 13, since there are 6 orders without ties, 1 with a 3-way tie, 3 with a 2-way ties for the first, and 3 with a 2-way tie for the last.)

12) Find A if there is a polynomial identity

$$(x^3 + 2x^2 - 3x - 2)(x^4 + 3x^3 + Ax^2 - 6x + 1) =$$
$$x^7 + 5x^6 + 5x^5 - 13x^4 - 23x^3 + 16x^2 + 9x - 2$$

Answers		
1)	2)	3)
4)	5)	6)
7)	8)	9)
10)	11)	12)

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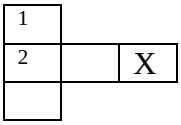
Round 2 (1 hour) **NO CALCULATORS School _____**

13) Three-digit powers of 2 and 5 are used in this cross-number puzzle. What is the only possible digit for "X"?

ACROSS DOWN
2. 2^m 1. 5^n



14) Find the largest positive value attained by the function

$f(x) = \sqrt{8x - x^2} - \sqrt{14x - x^2 - 48}$, x a real number.

15) In triangle ABC , $m\angle A = 55^o, m\angle C = 75^o$; D is on side $\overline{AB}$ and E is on side $\overline{BC}$. Find $m\angle BED$ if DB = EB.

16) The graphs of $y = -|x - a| + b$ and $y = |x - c| + d$ intersect at points (2, 5) and (8, 3). Find a + c.

17) What is the area of a triangle whose sides are 13, 13, and 10?

18) How many 2-digit numbers from 10 to 99 have the property that both digits are perfect squares? (i.e. 10 is the smallest such number and 99 is the largest)

19) How many ordered pairs (x, y) of integers satisfy $x^2 + 6x + y^2 = 16$?

20) Find the number that is halfway between 1/8 and 1/10.

21). You flip a fair coin repeatedly until either four consecutive Heads (H) or six consecutive Tails (T) occur. What is the probability that the sequence HHHH occurs before the sequence TTTTTT?

22) In triangle ABC, AB = 3, BC = 4, and AC = 6. If BC is extended through C to D so that CD = BC, what is AD?

23) Find the ordered pair of positive integers (a, b) with a < b, for which
$\sqrt{1+\sqrt{21+12\sqrt{3}}} = \sqrt{a} + \sqrt{b}$.

24) One angle of a triangle is twice another, and the sides opposite these angles have lengths 15 and 9. Compute the length of the third side of the triangle.

Answers		
13)	14) .	15)
16)	17)	18)
19)	20)	21)
22)	23)	24)