2015 CT-ARML runoff **PLEASE PRINT ALL INFO** **Name** ________________________

Round 1 (1 hour) **NO CALCULATORS** **School** _______________

1) Alphametic puzzles are arithmetic problems which involve words where each letter represents a unique digit that makes the equation true. If I represents the digit 0 and N represents the digit 4, what digit is represented by E?

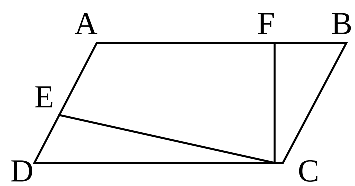
$$\begin{array}{cccccc} & O & B & A & M & A \\ + & B & I & D & E & N \\ \hline W & I & N & N & E & R \end{array}$$

2) A group of students takes a test and the average score is 74. If one more student had taken the test and scored 100, the average would have been 74.5. How many students took the test?

3) Triangle ABC has AC = 15, BC = 13, and AB = 4. What is the length of the altitude from C to the extension of AB?

4) During the lunch break from 12:00 TO 1:00, Bill eats, then checks his messages, then goes to the restroom, the talks to a friend. Each activity after the first takes half as much time as the preceding activity. There are no intervening time intervals. At what time did ill finish checking his messages?

5) In parallelogram ABCD, $\overline{CF} \perp \overline{AB}$ and $\overline{CE} \perp \overline{AD}$. If CF = 2, CE = 4, and FB is one fifth of AB, what is the ratio of the area of quadrilateral AFCE to that of parallelogram ABCD?



6) If $\log 650 = 2 - \log 2 + \log k$, compute k.

7) Let T = 40. If $x + 9y = 17$ and $Tx + (T+1)y = T + 2$, compute $20x + 14y$.

8) In a right triangle, one leg has length 8 and the other leg is 2 less than the hypotenuse. Compute the triangle's perimeter.

9) Charlie was born in the twentieth century. On his birthday in the present year (2014), he notices that his current age is twice the number formed by the rightmost two digits of the year in which he was born. Compute the four-digit year in which Charlie was born

10) A set S contains thirteen distinct positive integers whose sum is 120. Compute the largest possible value for the median of S.

11) What is the area of a triangle with sides $\sqrt{3}, \sqrt{4},$ and $\sqrt{5}$?

12) Three people play a game where the loser must double the money of the other two. After three games, each has lost once, and each has $32.00. How much did the person who lost first have at the start of the beginning of all the games?

Answers		
1)	2)	3)
4)	5)	6)
7)	8)	9)
10)	11)	12)

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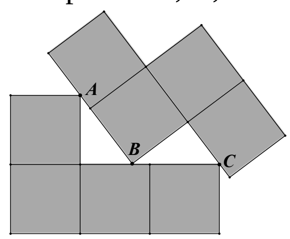
Round 2 (1 hour) **NO CALCULATORS** **School** _______________

13) If $a=\sqrt[10]{2}, b=\sqrt[50]{213},$ and $c=\sqrt[100]{2013}$, then arrange a, b, and c from smallest value to largest value.

14) Find the area of the region bounded by the graphs x – 4y = -8, x + y = -8, and 3x –2y = 6.

15) Eight unit squares are glued together to make two groups of four, and the groups are pressed together so as to meet in three points A, B, and C as shown in the diagram. Find the distance AB.



16) In octagon ABCDEFGH, sides AB, BC, CD, DE, and EF are part of a regular decagon, while sides FG, GH, and HA are part of a regular hexagon. What is the degree measure of angle CAH?

17) For what value of n is it true that $3^1 \cdot 3^2 \cdot 3^3 \cdots 3^n = 3^{253}$?

18) List all solutions *x* in [0, 2] to $\sin^2\left(x^2\right)=\frac{1}{4}$ where *x* is in radians.

19) List all real solutions *x* of the equation

$$(x+2)^{35}+(x+2)^{34}(x-1)^1+(x+2)^{33}(x-1)^2+\ldots+(x-1)^{35}=0.$$

20) The sum of all but one of the interior angles of a convex polygon is 2570°. Find the measure of the missing angle.

21) What is the solution for x > 0 of $\frac{1}{x+\sqrt{x}}+\frac{1}{x-\sqrt{x}}\le 1$?

22) In triangle ABC, P lies on AB with $\frac{AP}{PB}=\frac{1}{2}$, and N lies on AC with $\frac{AN}{NC}=3$. Let M be the midpoint of BC, and G the intersection of lines AM and PN. What is the ratio $\frac{AG}{GM}$?

23) Simplify completely:

$$\sin A \ne 0, \cos A \ne 0$$

$$\frac{\sin 3A}{\sin A}-\frac{\cos 3A}{\cos A}$$

24) Solve for θ: [0° ≤ θ < 360°]

$$2\sin 2\theta \cos 32^\circ + 2\cos 2\theta \sin 32^\circ = 1.$$

Answers		
13)	14) .	15)
16)	17)	18)
19)	20)	21)
22)	23)	24)