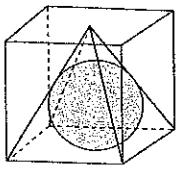


April 2, 2013
Round I: Arithmetic and Number Theory

1. (1 point) What is the units digit of $4(2)^{2013}$?

2. (2 points) Solve for n:

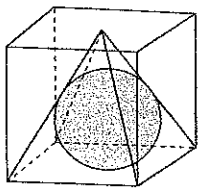
$$\frac{1+2+3+4+\ldots+n}{5n}=5.$$

3. (3 points) Andy plays a game where he wins if the sum of the numbers on two dice rolled is greater than or equal to 7. In order to increase his chances of winning, he secretly paints an extra dot on the 2 of one of the dice (turning it into a 3) and he paints an extra two dots on the 3 of the other die (turning it into a 5). What is the probability that Andy wins on one roll of these 2 dice?

1) _____

2) _____

3) _____



Round II
Algebra I (Real numbers and no transcendental functions)

1. (1 point) Solve for x: $|2x-3| = 9-2x$.

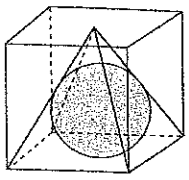
2. (2 points) I have some change in my pocket and the average value of the coins is 19 cents. If I had one more quarter then the average value would be 20 cents. How many dimes do I have? (No coin can be larger than a quarter.)

3. (3 points) The square root of $5+2\sqrt{6}$ has the simplified form $\sqrt{x}+\sqrt{y}$ where x and y are integers. Find the sum: x^2+y^2.

1) _____

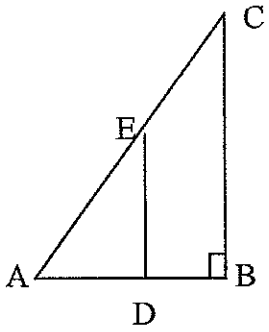
2) _____

3) _____



Round III
Geometry

1. (1 point) In ΔABC, $AB = 5$, $BC = 12$, and $m\angle ABC = 90°$. $\overline{ED}$ is parallel to $\overline{CB}$. $AE = \frac{3}{5}AC$. Find the area of quadrilateral DECB.



2. (2 points) Given circle O with diameter $\overline{AC}$. Chord $\overline{AB}$ is 3 units from center of the circle. If $m\angle BAC = 30°$, find the length of arc $\overset{\frown}{BC}$.

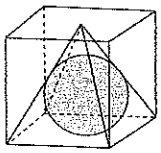
3. (3 points) A large circle has its center at A(1, -1) and is tangent to both the x and y axes. A small circle with center D is tangent to the large circle and both the x and y axes. What is the area of the small circle?

1) _____

2) _____

3) _____

Round IV
Algebra II



1. (1 point) Solve for x:
$$\log_3\left(x^2+5x-6\right)-2=\log_3(1-x).$$

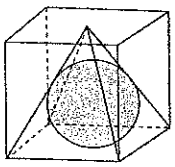
2. (2 points) Solve for x: $\dfrac{2x-6}{1-x}\le 2$.

3. (3 points) Find the remaining 3 roots of $10x^4-41x^3+60x^2-41x+10=0$ given that $\dfrac{4}{5}-\dfrac{3i}{5}$ is one root.

1) _____

2) _____

3) _____



Round V
Analytic Geometry

1. (1 point) A circle has center (8, 1) and is tangent to the line $y = -\frac{1}{2}x$. Find the coordinates of the point of tangency.

2. (2 points) A circle satisfies the following conditions:
   a) its center is at the focus of $y^2 - 4y + 4x = 0$
   b) the vertex of $y^2 - 4y + 4x = 0$ lies on the circle.

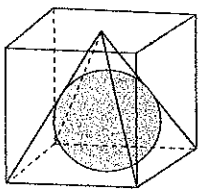
   Find the equation of the circle.

3. (3 points) If two points on the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are $\left(2\sqrt{2}, -\sqrt{3}\right)$ and $\left(\sqrt{10}, \frac{3}{2}\right)$, find the length of the major axis of the figure described by the equation.

1) _____

2) _____

3) _____



Round VI
Trigonometry, Complex Numbers

1. (1 point)
Find the value of $2010 \cdot i^{2010} + 2011 \cdot i^{2011} + 2012 \cdot i^{2012} + 2013 \cdot i^{2013}$.

2. (2 points) Solve for θ where $0° \le \theta < 360°$:

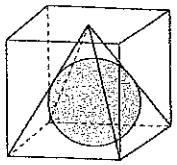
$$(\sin\theta + \cos\theta)^2 = \frac{1}{2}.$$

3. (3 points) One value of $Z^{1/3}$ is $2(\cos 50° + i \sin 50°)$ and $Z = a + bi$, where a and b are real numbers. Find the sum a + b.

1) _____

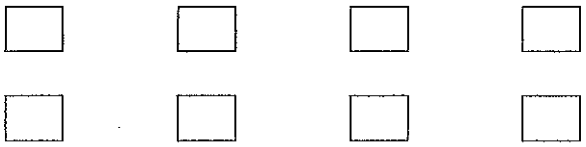
2) _____

3) _____



TEAM ROUND **NO CALCULATORS**

1) A small auditorium has 8 seats in the arrangement shown below.



In how many ways can the seats be occupied by 6 (different) boys and 2 (different) girls if each of the four corner seats must be occupied by boys?

2) The sum of 3 positive integers x, y, and z is 77. Taken in pairs, the respective differences of the integers are 4, 5, and 9. If $x < y < z$, find all possibilities for the 3 numbers.

3) In triangle ABC, $m\angle A = 60°$ and $m\angle B = 30°$. The bisectors of $\angle C$ and $\angle B$ meet at point E, and the line through B and E crosses $\overline{CA}$ at F, making FE:EB = 1:x. Find x.

4) Let f be a quadratic function for which $5f(x) - 3f(x-2) = 4x^2 + 4$ for all values of x. Find $f(3)$.

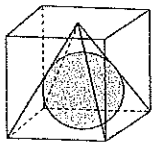
5) Square ABCD has sides of length 4, and M is the midpoint of $\overline{CD}$. A circle with radius 2 and center M intersects with a circle with radius 4 and center A at points P and D. What is the distance from P to $\overline{AD}$?

6) If x is an acute angle, then the inequalities

$$\sin x > \sqrt{1 - \sin 2x} \text{ and } \tan x > k$$

have exactly the same solution set. Find k.

CSAML Answers
Host: Bristol Eastern H.S.
April 2, 2013



Round I Arithmetic

1) 8

2) 49

3) 2/3

Round II Algebra I

1) 3

2) 2

3) 13

Round III Geometry

1) 96/5 or $19\frac{1}{5}$ or 19.2

2) 2π

3) $17\pi - 12\sqrt{2}\pi$ or $\pi\left(17 - 12\sqrt{2}\right)$

Round IV Algebra 2

1) -15

2) $x < 1$ or $x \geq 2$ OR $(-\infty, 1) \cup [2, \infty)$

3) $\left(\frac{4}{5} + \frac{3i}{5}\right), \frac{1}{2}, 2$

Round V Analytic Geometry

1) (6, -3)

2) $x^2 + (y-2)^2 = 1$

3) 8

Round VI Trigonometry & Complex

1) $2 + 2i$

2) 105°, 165°, 285°, 345°

3) $4 - 4\sqrt{3}$ or $4\left(1 - \sqrt{3}\right)$

TEAM Round

1) 8640

2) 21, 26, 30

3) $2 + \sqrt{3}$

4) 32

5) 16/5

6) 1/2