March 31, 2014
Round I: Arithmetic and Number Theory

1. (1 point)
How many palindromic numbers can be formed by putting any arrangement of digits from 0 to 9 into the five blank spaces of 2 _ _ 8 8 _ _ 4 _? (A palindromic number is a number that reads the same backward as forward. For example 27872.)

2. (2 points)
Determine the value of $\underline{a}$ in the decimal number 62894$\underline{a}$44 such that the entire number is divisible by 48.

3. (3 points)
The sales tax on an item is r%, where r is an integer, $0 < r < 50$. Let x be the price of an article in pennies, with $0 < x < 200$. If the article costs precisely 2 dollars when the unrounded sales tax(in pennies) is added, what is the cost in dollars of the item before the sales tax?

1) _____

2) _____

3) _____

Round II

Algebra I (Real numbers and no transcendental functions)

1. (1 point)

Solve for real x. $x-1+2\cdot 3=4\left[(5x-6)+7(8-9x)\right]$

2. (2 points) Given:

$f(x)=5x+a$

$h(x)=2x-b$

$f(h(x))=h(f(x))$ for all x.

Determine: $\frac{a}{b}$

3. (3 points) Solve for x:

$x+\frac{23}{5}\sqrt{x}=2$

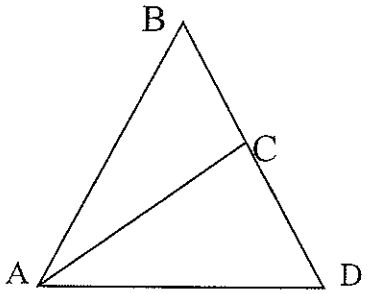
1) _____

2) _____

3) _____

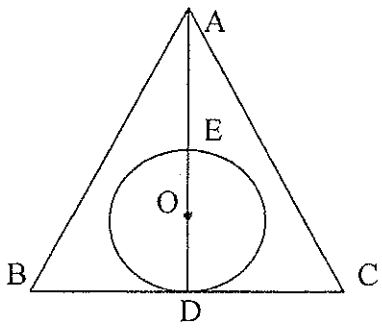
Round III
Geometry (figures are not to scale)

1. (1 point)
In triangle ABD, AB = AC = CD and AD = BD. Find $m\angle ADC$ (in degrees).



2. (2 points)
Quadrilateral QRST is inscribed in a circle. Given that $m\angle QTS = x^2 + 2x + 115$ degrees and $m\angle QRS = 3x + 71$ degrees, find all possible values for the measure of $\angle QTS$ in degrees.

3. (3 points)
ABC is an equilateral triangle of side 2. $\overline{AD}$ is an altitude of ΔABC, circle O is tangent to side $\overline{BC}$ at D and AD = 2AE. Determine the shortest distance from a point on circle O to $\overline{AC}$.



1) _____

2) _____

3) _____

Round IV
Algebra II .

1. (1 point) Factor completely:
$x^4 - 4x^3 + 4x^2 - 9y^2$

2. (2 points) Compute: $\sqrt{11+\sqrt{72}} + \sqrt{11-\sqrt{72}}$

3. (3 points)
If
$f(x) = 1 + x + x^2$
$g(x) = 2 + 3x + x^2$
$h(x) = 5 - x + 2x^2$
find constants a, b, c such that for all real values of x: $a(f(x)) + b(g(x)) + c(h(x)) = 2 - 8x + 3x^2$.
Express your solution as an ordered triple (a, b, c).

1) _____

2) _____

3) _____

Round V
Analytic Geometry

1. (1 point)
What is the area of the region enclosed by the graph of $|x-1|+|y+1|=2$?

2. (2 points)
Circle $(x-1)^2+(y-2)^2=4$ passes through the focal points of an ellipse whose major axis is parallel to the x-axis. The circle is also internally tangent to the ellipse. Determine the larger y-intercept of the ellipse.

3. (3 points)
The parabola with equation $y=ax^2+bx+c$ and vertex (h, k) is reflected about the line $y=k$. This results in a parabola with equation $y=dx^2+ex+f$.
What is the value of the sum $a+b+c+d+e+f$?

1) _____

2) _____

3) _____

Round VI
Trigonometry, Complex Numbers

1. (1 point)
Simplify: $\frac{i^{-5} - i^{24}}{i^{-7} + i}$

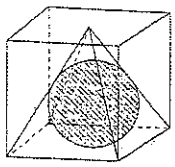
2. (2 points)
Compute the least positive degree measure for x for which
$8\sin x\cos^5 x - 8\sin^5 x\cos x = \sqrt{2}$

3. (3 points)
In a quadrilateral inscribed in a circle the sides are of length *a, b, c, d* in that order. Angle *K* is the angle between the two sides of length *a* and *b*. Find an algebraic formula for the cosine of angle *K* in terms of *a, b, c*, and *d*.

1) _____

2) _____

3) _____



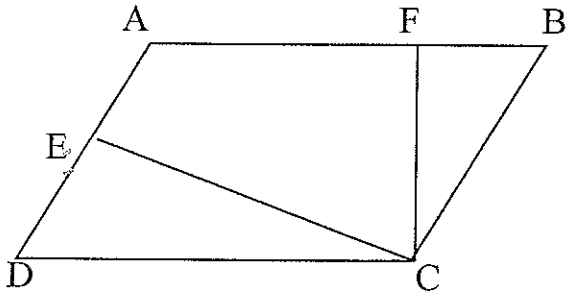
TEAM ROUND **NO CALCULATORS**

1) In the mini-Sudoku puzzle shown, each row of 4, each column of 4, and each of the 2 by 2 boxes must contain all the numbers 1, 2, 3, 4. The puzzle doesn't have enough information for a unique solution. Find the sum of all possible entries into the box labeled "x" that are part of a proper solution.

1			
			2
		3	
	x		

2) John ran an entire race in 50 minutes. The race was comprised of 3 distinct laps of equal length. He ran the first lap at an average speed of 12 km/hr. He ran each of the last two laps at an average speed of 16 km/hr. How many km long was the whole course? (total distance of the race)?

3) In parallelogram ABCD, $\overline{CF} \perp \overline{AB}$ and $\overline{CE} \perp \overline{AD}$. If CF = 2, CE = 4 and FB is one-sixth of AB, what fraction of the area of parallelogram ABCD is the area of quadrilateral AFCE?

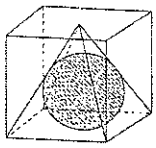


4) Solve for x: $2 = \sqrt{x+\sqrt{2x-1}} + \sqrt{x-\sqrt{2x-1}}$

5) A circle with center on the y-axis passes through the points (-7, -6) and (20, 3). The circle intersects the positive x-axis at $(a, 0)$. Find a.

6) Given right triangle PQR, right angle Q. $\frac{PQ}{QR} = \frac{\csc 78^\circ}{\sec 12^\circ}$ and $PR = \frac{1+\sqrt{2}}{5}$. Determine the area of ΔPQR.

CSAML Answers
Host: RHAM H.S
March 31, 2014



Round I Arithmetic

1) 10

2) 5

3) $1.60

Round II Algebra I

1) $\frac{195}{233}$

2) -4

3) $\frac{4}{25}$

Round III Geometry

1) 36°

2) 115°, 118°

3) $\frac{\sqrt{3}}{8}$

Round IV Algebra 2

1) $\left(x^2-2x-3y\right)\left(x^2-2x+3y\right)$

2) 6

3) (5, -4, 1)

Round V Analytic Geometry

1) 8

2) $\frac{4+\sqrt{14}}{2}$ or $2+\frac{\sqrt{14}}{2}$ or $\left(0,\frac{4+\sqrt{14}}{2}\right)$ or $\left(0,2+\frac{\sqrt{14}}{2}\right)$

3) 2k

Round VI Trigonometry & Complex

1) $-\frac{1}{2}+\frac{1}{2}i$ or $\frac{-1+i}{2}$

2) $\frac{45}{4}$

3) $\cos K=\frac{a^2+b^2-c^2-d^2}{2ab+2cd}$ or $\cos K=\frac{a^2+b^2-c^2-d^2}{2(ab+cd)}$

TEAM Round

1) 8

2) 12

3) $\frac{7}{12}$

4) 1.5

5) $\sqrt{301}$

6) $\frac{3+2\sqrt{2}}{100}$