CSAML

April 2, 2015
Round I: Arithmetic & Number Theory

1. (1 point) How many 2-digit prime numbers have the property that the sum of their digits is also a 2-digit prime number?

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2. (2 points) Find the sum of the units digit and tens digit of $(41)^{2015}$.

3. (3 points) Find the sum of the first 2015 odd integers.

1. _____

2. _____

3. _____

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April 2, 2015
Round II: Algebra I (Real numbers and no transcendental functions)

1. (1 point) What is the smallest integral value of k, such that $(2k-1)x^2 - 8x + 6 = 0$ has no real roots?

2. (2 points) For what real values of x is $\dfrac{1}{\sqrt{x+2} - \sqrt{2x-5}}$ a real number?

3. (3 points) For what real values of x is $\left(x^2 - 7x + 11\right)^{\left(x^2 - 3x - 18\right)} = 1$?
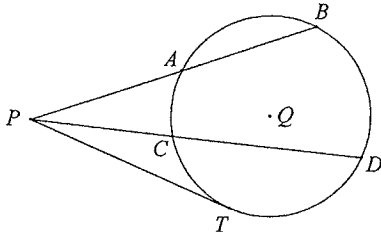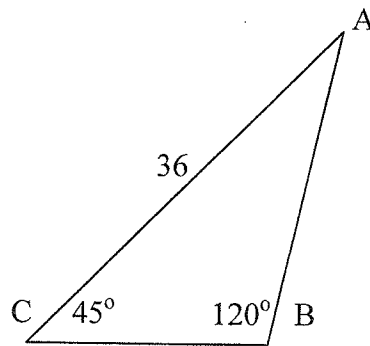
1. _____

2. _____

3. _____

April 2, 2015
Round III: Geometry (figures are not necessarily drawn to scale)

1. (1 point) If $\overline{PT}$ is tangent to circle Q and $PA = x+1, AB = 2x, PC = x-1,$ and $CD = 4x,$ find PT.

2. (2 points) Find the value(s) of x which would allow the three lengths $2x - 3, x+6,$ and $3x+1$ to be the sides of a triangle.
(The area of the triangle must not be zero.)

3. (3 points) Find the area of $\triangle ABC$.

1. _____

2. _____

3. _____

April 2, 2015
Round IV: Algebra II

1. (1 point) Solve for x: $\log_8 3 = x \log_2 3$.

2. (2 points) Solve for a: $a^{\frac{3}{2}} + 216 = 35 a^{\frac{3}{4}}$.

3. (3 points) If $\dfrac{x}{2-y} = \dfrac{y}{4-z} = \dfrac{z}{8-x} = 3$, compute $x+y+z$.

1. _____

2. _____

3. _____

April 2, 2015
Round V: Analytic Geometry

1. (1 point) Find the coordinates of the vertices of the hyperbola with equation
$y^2 - 5x^2 - 6y - 70x - 241 = 0$.

2. (2 points) A parabola opens down and has vertex at P(2, 3). Point Q(0, 1) is a point on the parabola. Line $\overleftrightarrow{RS}$ intersects the parabola at x = 4 and x = -4. Determine the coordinates of the y-intercept of $\overleftrightarrow{RS}$.

3. (3 points) Find the radius of the largest circle whose center is in the first quadrant and is tangent to both the x- and y-axis and contains the point (4, 8).

1. _____

2. _____

3. _____

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1. (1 point) Let $\dfrac{-\pi}{2} < x < 0$ and $\sin x = \dfrac{-3}{5}$. Let $0 < y < \dfrac{\pi}{2}$ and $\tan y = \dfrac{5}{12}$.

Evaluate $\cos(x+y)$.

2. (2 points) Given: $\left(2^{\sin^2 x}\right)\left(2^{\cos^2 x}\right)\left(2^{\tan^2 x}\right) = \sqrt[3]{32}$. If x is the least positive angle for which the above statement is true, what is the value of $\sec x$?

3. (3 points) Find x as a function of θ so that $\tan\theta\left(x + \cot\theta\cos\theta\right) = \sec\theta$ for all θ for which the given trig functions are defined.

1. _____

2. _____

3. _____

Team Round   April 2, 2015

1. Math High belongs to a 6 school league in which each school plays four games with each of the other schools. No tied games ever occur, and the other five schools finished the season having won respectively, 20%, 30%, 35%, 60%, and 80% of the league games they played. What was Math High's final winning record in the league this season? (Express as a percent)

_____

2. Line ℓ is perpendicular to the line $\overline{AB}$ and passes through the midpoint of segment $\overline{AB}$. If A, B are the intercepts of the line $3x - 5y = c$ find, in terms of c, the y-intercept of the line ℓ.

_____

3. On the x-axis, the diameter of the circle $x^2 + y^2 = 1$ is $\overline{AB}$ and the diameter of the circle $(x-1)^2 + y^2 = 4$ is $\overline{AC}$. The unit of measure is yards. Traveling at a constant speed, a model train starts at C, travels counter-clockwise along the circle to A, goes around the inner circle and back to C in 25 seconds. What is the speed in feet per minute?

_____

4. Factor completely: $x^6 - 35x^4 + 259x^2 - 225$

_____

5. Find the area of the region defined by the graphs: $|x+y| \le 2$ and $|x| + |y| \le 6$.

_____

6. If x is a real number between 0 and $\frac{\pi}{4}$ and if $\sin x + \cos x = \frac{2\sqrt{3}}{3}$, find the exact value of $\sin x - \cos x$.

_____

CSAML Answers for 2015 Meet
Round I

1) 5

2) 1

3) 4,060,225

Round II

1) 2

2) $x \neq 7$ and $x \geq 2.5$ or $\left[\dfrac{5}{2}, 7\right) \cup (7, \infty)$

3) $x = -3, 2, 3, 4, 5, 6$

Round III

1) $4\sqrt{6}$

2) $x \in (2, \infty)$ or $x > 2$

3) $324 - 108\sqrt{3}$ or $108\left(3 - \sqrt{3}\right)$

Round IV

1) $\dfrac{1}{3}$

2) 16, 81

3) $\dfrac{21}{2}$

Round V

1) $\left(-7, 3 + \sqrt{5}\right), \left(-7, 3 - \sqrt{5}\right)$

2) $(0, -7)$

3) 20

Round VI

1) $\dfrac{63}{65}$

2) $\dfrac{\sqrt{15}}{3}$

3) $\sin \theta$

TEAM Round

1) 75%

2) $\dfrac{8c}{45}$ or $\left(0, \dfrac{8c}{45}\right)$

3) $\dfrac{216\pi}{5}\left(\dfrac{ft}{\min}\right)$

4) $(x+1)(x-1)(x+3)(x-3)(x+5)(x-5)$

5) 24

6) $\dfrac{-\sqrt{6}}{3}$