CSAML 2016

April 6, 2016
Round I: Arithmetic & Number Theory

1. (1 point) The mean score on a test taken by 21 students on one day was 82. Three students were absent on the day of test and took it the next day. These 3 students did so well that the mean score for all 24 students was raised 1(one) point. What was the mean score for these 3 absent students?

2. (2 points) If $\dfrac{1}{a+\sqrt{b}} = a-\sqrt{b}$ with a, b are positive integers and $a < 10$, determine how many ordered pairs (a, b) exist, such that a + b is prime.

3. (3 points) A and B are integers greater than 1 such that $A^9B^3 = (57)^3(117)^3$. Find $A+B$.

1. ______________________________

2. ______________________________

3. ______________________________

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Round II: Algebra I (Real numbers and no transcendental functions)

1. (1 point) Find all values of a such that $|a-4|^2 - |a-4| = 6$.

2. (2 points) Evaluate: $x^3 - x^2y - xy^2 + y^3$ if $x = 2015$ and $y = 2016$.

3. (3 points) When $kx^2 + 5x + 6$ is divided by $x + 2$, the remainder is the same as when $kx^2 + 5x + 6$ is divided by $x - 3$. Find all possible values of k.

1. ______________________________

2. ______________________________

3. ______________________________

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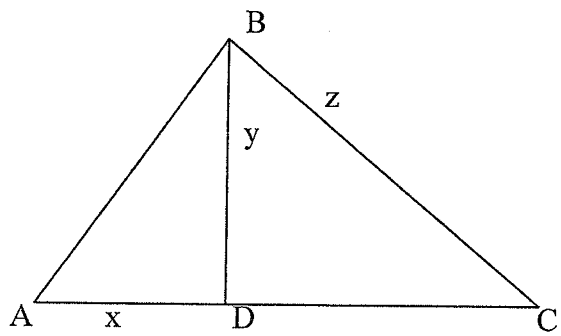
Round III: Geometry (figures are not necessarily drawn to scale)

1. (1 point) How many circular pipes with an inside diameter of one inch would be needed to carry the same amount of water as a single circular pipe with an inside diameter of one foot?

2. (2 points) Triangle ABC, shown below, has a right angle at B. Segment BD is an altitude of the triangle. AB = 4 and CD = 6. Compute x + y + z.



3. (3 points) A circle is inscribed in a 60° sector of a larger circle. The smaller circle is internally tangent to the larger circle, and is tangent to the two radii that form the sector. What is the ratio of the radius of the smaller circle to the radius of the larger circle?

1. ______________________________

2. ______________________________

3. ______________________________

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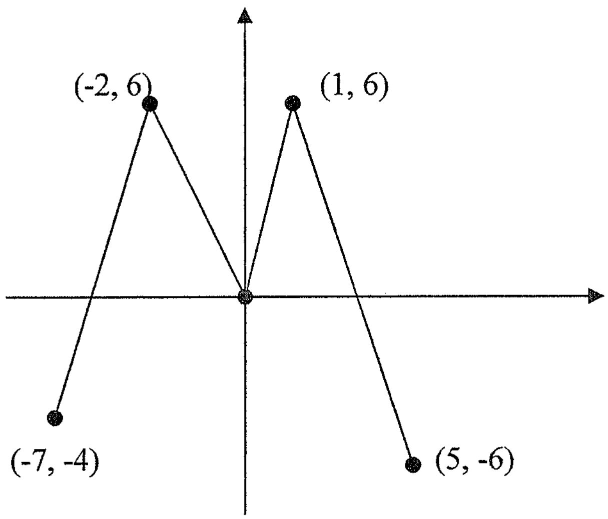
Round IV: Algebra II

1. (1 point) A function f is a given by $f(x)=\dfrac{4x+3}{x}$. Find the value of a such that $f(2a)=2f(a)$.

2. (2 points) The graph of the function f is shown below. How many solutions does the equation $f(f(x))=6$ have?



3. (3 points) Find, in terms of b, the remainder when x^3+bx+b is divided by $x-b+1$.

1. ______________________________

2. ______________________________

3. ______________________________

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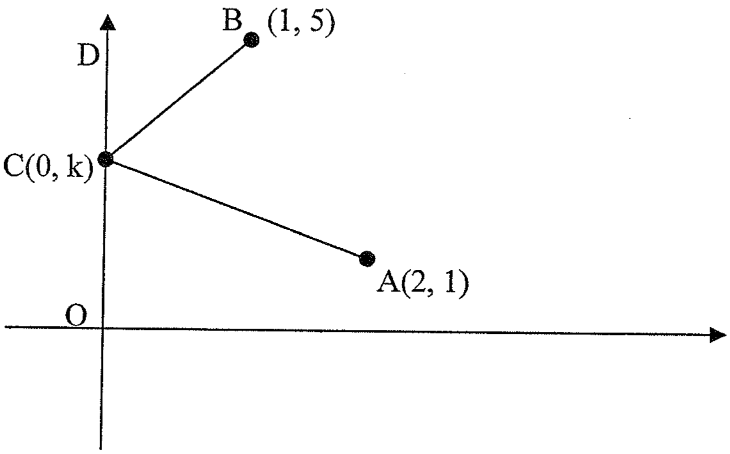
Round V: Analytic Geometry

1. (1 point) Refer to the diagram below to find k so that $\angle DCB \cong \angle ACO$.



2. (2 points) Find the distance between the vertex of $f(x) = 2x^2 + 16x + 35$ and the vertex of its inverse relation.

3. (3 points) Find the equation of the common chord of the two circles

$(x+1)^2 + (y-3)^2 = 7$ and $\left(x - \frac{3}{2}\right)^2 + (y+4)^2 = \frac{169}{4}$.

Leave your answer in the form Ax + By + C = 0, where A, B, C are integers with no common factor other than 1, and A > 0.

1. ______________________________

2. ______________________________

3. ______________________________

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Round VI: Trigonometry

1. (1 point) An ant is standing on horizontal ground. When the ant looks at the top of a building, the angle of elevation is 60°. The height of the building is 30 meters. How far, in meters, is the top of the building from the ant?

2. (2 points) Find the value of $\tan x$ if $\dfrac{\cos(x-60^o)}{\sin x} = 2$.

3. (3 points) The three solutions of the equation $x^3 = 8i$ are $a_1 + b_1 i,\ a_2 + b_2 i,\ a_3 + b_3 i$, where $a_1, a_2, a_3, b_1, b_2, b_3$ are real. Find $b_1^2 + b_2^2 + b_3^2$.

1. ______________________________

2. ______________________________

3. ______________________________

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Team Round

April 6, 2016

1. On the planet Mathlandia there are as many days in a week as there are weeks in a month. The number months in a year is half the number of days in a week. If there are 864 days in a Mathlandia year how many days are in a Mathlandia week. (Note: The number of months in the year is an integer, and the months are of equal length.)

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2. Solve for x: $$\frac{2-\frac{4}{x}}{\left(1+\frac{3}{x}\right)\left(1-\frac{5}{x}\right)}=\frac{1-\frac{8}{x}}{1-\frac{5}{x}}+1.$$

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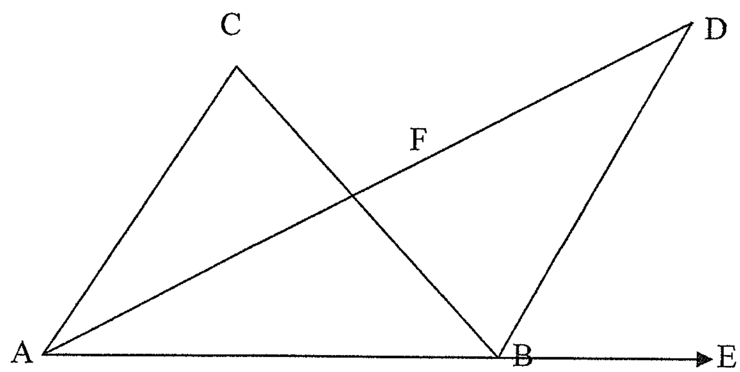
3. Given:

$m\angle D = 40^{o}$

$\overrightarrow{AF}$ bisects $\angle CAB$

$\overrightarrow{BD}$ bisects $\angle CBE$

Determine: $m\angle C$



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4. Solve the following system of equations for x and y.

$$\begin{cases}\log_2(x+1)-\log_2 y=1\\ \log_3(x+y)=\log_3 x+\log_3 y\end{cases}$$

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5. Find the area of the plane region formed by the intersection of the graphs

$$\begin{cases} y \geq |x+2| \\ |y-5| \leq 1 \end{cases}.$$

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6. Solve, for $0 < x < \pi$, the equation $\csc x - \cot x = \sqrt{3}$.

GIVE YOUR SOLUTION(S) IN RADIANS.

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Answers

Round 1

1) 90

2) 7

3) 250

Round 2

1) 1 or 7 accept 1,7
Accept 1 and 7

2) 4031

3) -5

Round 3

1) 144

2) $2+6\sqrt{3}$

3) 1 : 3 OR $\frac{1}{3}$

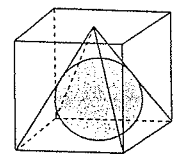
Round 4

1) $-\frac{9}{8}$

2) 6

3) b^3-2b^2+3b-1

Round 5



1) $\frac{11}{3}$

2) $7\sqrt{2}$

3) $5x-14y+27=0$

Round 6

1) $20\sqrt{3}$

2) $\frac{4+\sqrt{3}}{13}$

3) 6

Team

1) **12**

2) -13

3) 80

4) $x=1+\sqrt{2}, y=\frac{2+\sqrt{2}}{2}$

5) 20

6) $x=\frac{2\pi}{3}$