2012 CT-ARML runoff **PLEASE PRINT ALL INFO** **Name** ________________________

Round 1 (1 hour) **NO CALCULATORS** **School** _______________

1) The perimeter of a rectangle is 28. A second rectangle is three times as long as the first, and twice as wide. The perimeter of the second rectangle is 72. What is the area of the first rectangle?

2) What is the solution set of the inequality $x^3 + x^2 - 2x \geq 0$?

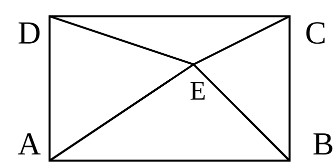
3) Let *a*, *b*, *c*, *d*, and *e* be distinct integers such that (6-*a*)(6-*b*)(6-*c*)(6-*d*)(6-*e*) = 45. What is *a* + *b* + *c* + *d* + *e*?

4) Triangles ABC and ADC are isosceles with AB = BC and AD = DC. Point D is inside triangle ABC, $m\angle ABC = 40^o$, and $m\angle ADC = 140^o$. What is the degree measure of $\angle BAD$?

5) At a twins and triplets convention, there were 9 sets of twins and 6 sets of triplets, all from different families. Each twin shook hands with all the twins except his/her sibling and with half the triplets. Each triplet shook hands with all the triplets except his/her siblings and with half the twins. How many handshakes took place?

6) Suppose that $\sin a + \sin b = \sqrt{5/3}$ and $\cos a + \cos b = 1$. What is $\cos(a - b)$?

7) In the diagram, ABCD is a rectangle, and E is in the interior of the rectangle. If AE = 7, BE = 6, and CE = 4, find DE.



8) The product of the first five terms of a geometric progression is 32. If the fourth term is 17, compute the second term.

9) Three circles are mutually externally tangent. The large circle has a radius of T, and the smaller two circles have radius T/2. Compute the area of the triangle whose vertices are the centers of the three circles.

10) Compute the value of x that satisfies $\sqrt{20 + \sqrt{11 + x}} = 5$.

11) In Δ ABC, the ratio of side $\overline{BC}$ to side $\overline{AC}$ is $\sqrt{10} : \sqrt{15}$. If A = Arctan 1, find the measure of $\angle C$ of the triangle in degrees.

12) Evaluate as a simple fraction $\sum_{i=1}^{100} \frac{1}{4i^2 - 1}$

Answers

1)	2)	3)
4)	5)	6)
7)	8)	9)
10)	11)	12)

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Round 2   (1 hour)   **NO CALCULATORS**   **School** ______________

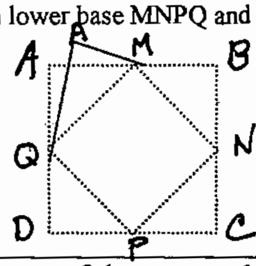
13) What is the largest integer n such that 20! Is divisible by 80^n?
(Note $20! = 1 \cdot 2 \cdot 3 \cdots 20$)

14) Harry is somewhere between his home and the football stadium. To get to the stadium he can walk directly to the stadium, or else he can walk home and then ride his bicycle to the stadium. He rides 7 times as fast as he walks, and both choices require the same amount of time. What is the ratio of Harry's distance from his home to his distance from the stadium?

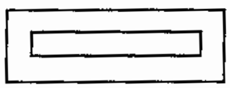
15) Given that $\frac{((3!)!)!}{3!} = k \cdot n!$, where k and n are positive integers and n is as large as possible, find k + n.

16) Triangles ABC and ADC are isosceles with AB = BC and AD = DC. Point D is inside triangle ABC, $m\angle ABC = 40°$, and $m\angle ADC = 140°$. What is the degree measure of $\angle BAD$?

17) ABCD is a square with M, N, P, and Q midpoints of sides as shown. A fold is made along QM and plane AMQ is perpendicular to MNPQ. The same is done with B, C, and D. Points A, B, C and D are joined in succession forming another square ABCD. Thus, an open-top solid is formed with lower base MNPQ and upper base ABCD. Find the volume of the solid if the original AB = 4.



18) Mildred prefers her brownies from the center of the pan, and Millicent prefers them from around the edge. If they bake a 9 by 12 pan of brownies, how far from the edges of the pan should they cut so that each get equal areas of brownies?



19) The number $(8\ 1\ 1\ a)_{nine}$ (where this is a base 9 numeral) is a perfect square. What is the value of a?

20) Circle X_1 has center O which is on circle X_2. The circles intersect at points A and C. Point B lies on X_2 such that BA = 37, BO = 17, and BC = 7. Compute the area of X_1.

21) Compute all ordered pairs (x, y) such that $\begin{cases} xy + 9 = y^2 \\ xy + 7 = x^2 \end{cases}$.

22) Two fair coins are flipped simultaneously. This is done repeatedly until at least one of the coins comes up heads, at which point the process stops. What is the probability that both coins came up heads on this last flip?

23) Find all ordered pairs (s, y) of positive real numbers such that 3, x, y is a geometric progression, while x, y, 9 is an arithmetic progression.

24) Find the equation of the circle tangent to the line L with equation 3x + 2y + 7 = 0 at the point T(-1, -2) and with its center on the line 8x – 5y + 5 = 0.

Answers		
13)	14)	15)
16)	17)	18)
19)	20)	21)
22)	23)	24)