2017 CT ARML Runoff Part I (1 hour) Please print all information legibly

Name _____ School _____ Grade _____

1) Let A, B, and C be digits in base 7, with possible values 0, 1, ..., 6. If $ACB_7 + BCC_7 = 1400_7$, what is the base 10 value of ACB_7?

2) Find the solution set for the inequality: $\frac{1}{x} + 2x \geq 3$.

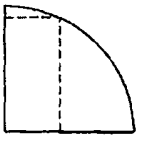
3) A triangle has a right angle at A with AC = 3 and AB = 1. The angle bisector at B meets AC at P. What is the length of CP?

4) A function f satisfies $f(0) = 0$, $f(2n) = f(n)$, $f(2n+1) = f(n) + 1$ for all positive integers n. What is the value of $f(2017)$.

5) A cube of cheese $c = \{(x,y,z) | 0 \leq x,y,z \leq 1\}$ is cut along the planes $x = y$, $y = z$, and $x = z$. How many pieces are there?

6) What is the area of a triangle with sides 10, 10, 16?

7) A rectangle is inscribed in a quarter-circle of radius 6, as shown, so that The sum of the width and height is 8. What is the area of the rectangle?



8) Find the number of integers, $n, 1 \leq n \leq 25$ such that $n^2 + 3n + 2$ is divisible by 6.

9) The longer leg of a right triangle is equal to the hypotenuse of a 30°-60°-90° triangle. If the two triangles have equal perimeters, what is the tangent of the smallest angle of the first triangle?

10) Three vertices of a cube are P = (7, 12, 10), Q = (8, 8, 1) and R = (11, 3, 9). What is the surface area of the cube?

11) If $x = \log(8)$ and $y = \log(9)$, then express $\log(120\sqrt{2})$ in terms of x and y.

12) How many ways can we obtain \$20.15 using ***only*** quarters and dimes?

Answers		
1) 319	2) $(0,1/2] \cup [1,\infty)$	3) $(10-\sqrt{10})/3$
4) 7	5) 6	6) 48
7) 14	8) 13	9) $(3-\sqrt{3})/4$
10) 294	11) $1 + \frac{5}{6}x + \frac{1}{2}y$	12) 40

No Calculators 1 hour time limit

2017 CT ARML Runoff Part II (1 hour) Please print all information legibly

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13 Compute the least possible, non-zero value of $A^2 + B^2 + C^2$ such that A, B, and C are integers satisfying $A\log 16 + B\log 18 + C\log 24 = 0$.

14) How many ordered pairs (x, y) of integers (not necessarily positive) satisfy $\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$?

15) What is the smallest integer larger than $\left(\sqrt{5} + \sqrt{3}\right)^6$?

16) The magic square shown uses each integer from 1 through 9, exactly once, so that The sum along any row, column, and both diagonals is 15. What is the value of x?

	9	4
x		

17) The price of a shirt is increased 25%, and then there is another increase of 20%. What is the overall percentage increase?

18) In this addition example, AA + BB + CC = BAC different letters represent different digits. What is the value of the three-digit number BAC?

19) There are 10 Bluray's in a package. Mike reads the front of the package and realizes that 3 of the Bluray's are ones he likes. If he selects 4 at random, what is the probability that he gets exactly two of the ones he likes?

20) If $\sin x + \cos x = \sin x \cos x$, then what is $\sin x \cos x$?

21) If $f(x) = x^2 + 1$, what is the value of $f\left(f\left(f\left(f\left(0\right)\right)\right)\right)$?

22) If a and b are positive real numbers satisfying $\left(a-b\right)^2 = 4\left(ab\right)^3$, what is the smallest possible value of $\frac{1}{a} + \frac{1}{b}$?

23) Triangle ABC has AB = 6, AC = 5 and BC = 4. Points P_1, P_2, and P_3 on BC satisfy $BP_1 = BP_2 = BP_3 = P_3C = 1$. What is the value of $\left(AP_1\right)^2 + \left(AP_2\right)^2 + \left(AP_3\right)^2$?

24) Compute the largest of the three prime divisors of $13^3 + 16^5 - 172^2$.

Answers		
13) 105	14) 9	15) 3904
16) 7	17) 50	18) 198
19) 3/10	20) $1-\sqrt{2}$	21) 26
22) $2\sqrt{2}$	23) 163/2	24) 1321

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