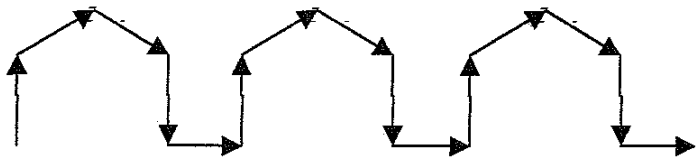
CSAML

~~April 3, 2010~~ 2012
Round I: Arithmetic and Number Theory

1) Sketch the 2012th arrow of the sequence:



2) Let the sum, difference, and the product of two numbers be S, D and P, respectively. The ratio S:D:P is 11:1:90. Find P.

3) What is the sum of the last two digits of the number named by $\left(4^3\right)^{2012}$?

1) _____

2) _____

3) _____

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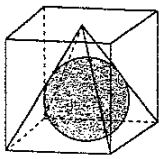


Round II
Algebra I

1) Solve: $\left|\left|x-1\right|-1\right|-1=1.$

2) Solve for (x, y, z)

$$\begin{cases} \frac{3}{x}+\frac{1}{y}=61 \\ \frac{3}{y}-\frac{1}{z}=1 \\ \frac{1}{x}+\frac{3}{z}=26 \end{cases}$$

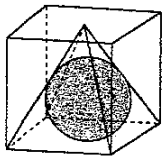
3) Solve: $\sqrt[3]{x^2+16x+64}-2\sqrt[3]{x+8}-24=0.$

1) _____

2) _____

3) _____

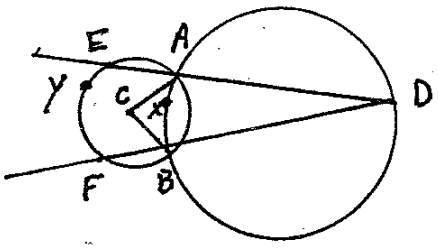
CSAML



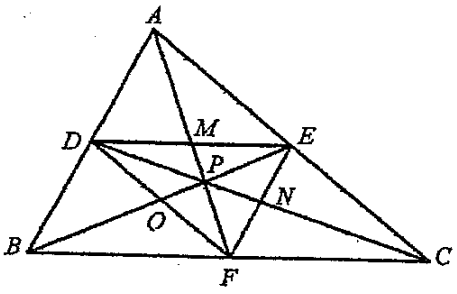
Round III
Geometry

1) In a right triangle, the hypotenuse has a length two more than the longer leg and a length one more than 5 times the shorter leg. What is the length of the hypotenuse?

2) Let C be the center of the circle. If $m\angle ACB = 40°$ and $m\overset{\frown}{EYF} = 60°$, find $m\overset{\frown}{AXB}$.



3) In ΔABC, the sum of the lengths of the medians $\overline{AF}, \overline{BE}$, and $\overline{CD}$ is 108. Find the value of $MP + NP + OP$.



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1) _____

2) _____

3) _____

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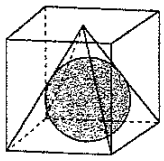

Round IV
Algebra II

1) Find the domain of the function $f(x) = \sqrt{1 - \ln x}$.

2) When $ax^{2012} - bx^{2011} + 2010$ is divided by $x+1$, the remainder is 5. When the same expression is divided by $x-1$, the remainder is 15. Find the ordered pair (a, b).

3) For what values of a will the slope of the line containing the points (5, a) and (a, -2) be greater than or equal to 2?

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1) _____

2) _____

3) _____

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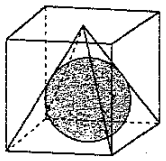


Round V
Analytic Geometry

1) The line joining the vertices of a hyperbola is parallel to the x-axis and the center of the hyperbola is at (5, 3). The hyperbola passes through the origin and the point (7, 3). The equation of the hyperbola can be written in the form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, where $a > 0$ and $b > 0$. Find the value of b.

2) Find coordinates of the endpoints of the major axis if the conic section with equation $3x^2 + 2y^2 - 18x + 16y + 58 = 0$.

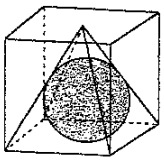
3) An isosceles triangle with base endpoints (-4, -3) and (-3, 4) is inscribed into the circle $x^2 + y^2 = 25$. What are the exact possible coordinates of the vertex angle of this triangle?

1) _____

2) _____

3) _____

CSAML



Round VI
Trigonometry, Complex Numbers

1) Solve for x, $0 \le x < 2\pi$

$$\frac{\tan x}{2\sin x} = -1.$$

2) Regular dodecagon ABCDEFGHIJKL has perimeter 108. Find the square of the distance between points F and H.

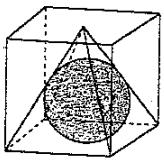
3) For what values of x, $0 \le x < 2\pi$ will $3 + 4\cos(3x)$ take its minimum value?

1) _____

2) _____

3) _____

CSAML



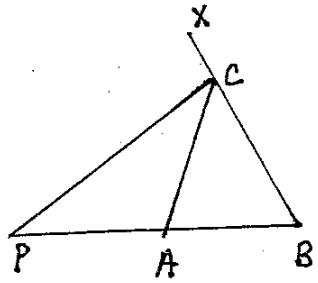
TEAM ROUND

1) The first 100 numbers of the sequence 9, 99, 999, 9999, ... are added together. If a b c d are the last four digits of the sum, find a b c d..

2)

Solve for x: $\frac{b+x}{b}+\frac{2x}{b+x}+\frac{x^2}{b^2-bx}=2$

3) In ΔABC (see the diagram) CA = 3 and CB = 4. PAB is a straight line and PC bisects angle ACX. Find the ratio PA:AB

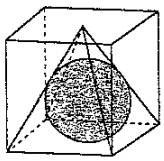


4) Factor completely over the integers: $27x^3-y^3+3y^2-3y+1$.

5) Find the area of the polygon formed by the intersections of the graphs of

$$\begin{cases}(x-3)^2+(y+2)^2=289\\ y+19=\frac{1}{2}(x-3)^2\end{cases}$$

6) P(x) is a polynomial of least degree with integral coefficients (include the constant) with greatest common factor of 1. Find the sum of these coefficients (include the constant) if two of the roots of P(x) are $\frac{1}{2}+\frac{i\sqrt{7}}{3}$ and $3-2i$.



CSAML Answers
Host: E.O.Smith High School
April 3, 2012

Round I Arithmetic

1. ↗

2. 270

3. ~~96~~ 15

Round II Algebra I

1. $x = 4, x = -2, x = 2, x = 0$

2. $\left(\frac{1}{20}, 1, \frac{1}{2}\right)$

3. $x = 208, x = -72$

Round III Geometry

1. 101

2. 20

3. 18

Round IV Algebra 2

1. $0 < x \leq e$

2. (-2000, -5)

3. $a \in \left[\frac{8}{3}, 5\right)$

Round V Analytic Geometry

1. $\frac{2\sqrt{21}}{7}$

2. $\left(3, -4 \pm \frac{\sqrt{2}}{2}\right)\left(3, -4 \pm \frac{\sqrt{2}}{2}\right)$

3. $\left(\frac{7\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right), \left(-\frac{7\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

Round VI Trigonometry & Complex

1. $\frac{2\pi}{3}, \frac{4\pi}{3}$

2. $81\left(2+\sqrt{3}\right) = 162 + 81\sqrt{3}$

3. $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

TEAM Round

1. 1010

2. $\frac{b}{3}$

3. 3:1

4. $(3x - y + 1)(9x^2 + 3xy - 3x + y^2 - 2y + 1)$

5. 256

6. ~~-185~~ 296