NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2012

Round 1: Arithmetic and Number Theory

1. $a=$______ ; $b=$________

2. ________________

3. ________________

1. Find a and b, relatively prime, such that $\dfrac{0.\overline{36}\cdot 1.\overline{2}}{0.\overline{07}}=\dfrac{a}{b}$

2. How many times in this century is the sum of the digits of the year a perfect square?

3. Bob was numbering the pages of a book, but he made a mistake. After writing page number 169, he then wrote 160 for the next page and continued numbering the remaining pages consecutively based on that mistake. Without making any additional mistakes, he finished at page 220. If he can't change the order of the pages, what is the fewest number of digits that he can erase and replace to obtain a correct numbering scheme?

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Round 2: Algebra 1

1. _______________ %

2. _________:_________

3. ____________________

1. Gina was given a raise of 25%. She felt pretty good. But then the next week she was told that her salary was being cut by 25%. She was upset. By what percent does her latest salary differ from her salary before the raise?

2. Compute the ratio of the positive root of $9x^2 + 6x - 1 = 0$ to the positive root of $x^2 + 2x - 1 = 0$. Express the answer as a ratio of whole numbers.

3. Compute the <u>largest</u> of the three positive reals a, b, and c such that

$$ab = 100$$
$$bc = 150$$
$$ac = 135$$

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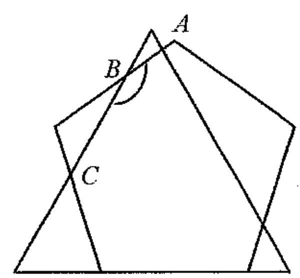
Round 3: Geometry

1. ______________________

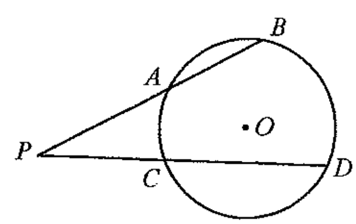
2. ______________________

3. ______________________

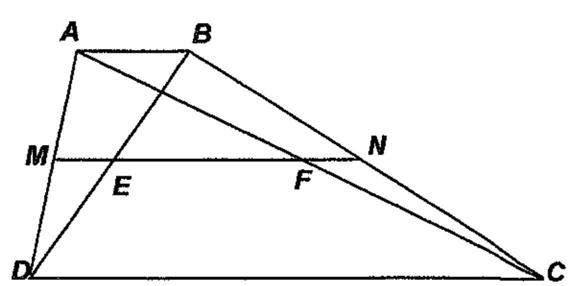
1. The diagram shows an equilateral triangle and a regular pentagon. A side of the pentagon lies on a side of the triangle. What is the degree-measure of angle ABC?



2. From point P secants $\overline{PB}$ and $\overline{PD}$ are drawn to circle O. If the lengths of $\overline{PA}$, $\overline{AB}$, $\overline{PC}$, and $\overline{CD}$ are selected from $\{5, 6, 7, 8\}$, without replacement, what are the possible lengths of $\overline{AB}$?



3. In trapezoid $ABCD$, $\overline{AB} \parallel \overline{CD}$, $\overline{MN}$ is a median, and both AB and DC are integers with $AB < DC$ and $AB + DC = 48$. If $EF > AB$, find the number of ordered pairs (AB, DC).



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Round 4: Algebra 2

1. ______________________

2. ______________________

3. ______________________

1. If $\log_b 2 = m$, $\log_b 9 = n$, and $\log_b 125 = p$, determine $\log_b\left(1\frac{23}{27}\right)$ as a single reduced fraction in terms of *m*, *n*, and *p*.

2. If the real solutions of the equation $(x+8)^4 + 5x^2 + 80x + 320 = 14$ are denoted by *m* and *n*, determine $|m-n|$.

3. The real roots of $x^3 - ax^2 + nax - k = 0$ form a geometric sequence; *a*, *n*, and *k* are real numbers; $a \neq 0$, $n \neq 1$ and $k \neq 0$. If the product nk equals n^t, find *t*.

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Round 5: Analytic Geometry

1. ____________________

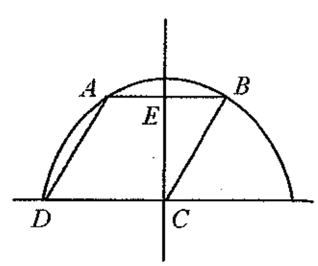
2. ____________________

3. ____________________

1. An ellipse whose axes are either vertical or horizontal is centered at the origin. The graph of $2x - 7y = 14$ passes through an endpoint of the major axis and an endpoint of the minor axis. What is the smallest distance between a focus and one end of the major axis?

2. Given $M(2a+1, 3b+2)$, $N(5a+2, 6b+1)$, and $P(8a+3, 8b)$, with a an integer from −5 to 5, inclusive. If N is the midpoint of non-vertical segment $\overline{MP}$, find the slope for $\overline{MP}$ that is numerically the largest.

3. *ABCD* is a rhombus. Points *A*, *B* and *D* lie on a parabola whose equation is $f(x) = ax^2 + k$, *C* lies on the origin and *E* lies on the *y*-axis. If $B = (p, f(p))$, compute the numerical value of the product ap.



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Round 6: Trig and Complex Numbers

1. ____________________

2. ____________________

3. ____________________

1. Let A = least positive value of y where $y = 5\sec x - 2$ for x in radians. Find, in radians, $\mathrm{Tan}^{-1}\sqrt{A} + \mathrm{Cos}^{-1}\dfrac{\sqrt{A}}{2}$.

2. Determine the number of solutions in $[0, 2\pi)$ to $\dfrac{\sin 5x}{\sin x} + \dfrac{\cos 5x}{\cos x} = 0$.

3. If $\sin x \cos x = \dfrac{1 - \cos 4x}{2} = k$, for $k > 0$, find k.

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Team Round

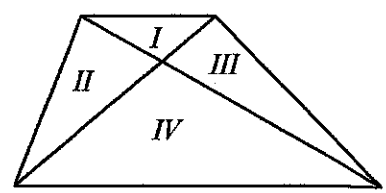
1. ____________________ **4.** ____________________

2. ____________________ **5.** ____________________

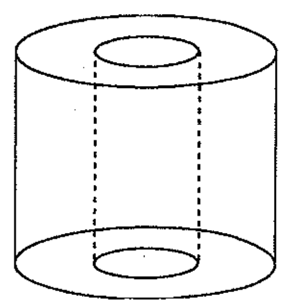
3. ____________________ **6.** ____________________

1. In how many ways can 6 different people sit in a row of 6 chairs if person A refuses to sit in the first chair and person B refuses to sit in the last chair?

2. In the trapezoid the Roman numerals *I*, *II*, *III*, and *IV* represent the areas of the four triangular regions that make up the trapezoid, with *IV* being the largest. If *I*, *II*, and *III* are integers and $IV = 256$, find the largest possible area of the trapezoid.



3. Starting with a right circular cylinder of height 12 and radius r, a hole is drilled completely through the cylinder and both bases, creating a pipe whose internal radius is x. If the total surface area of the new figure equals the total surface area of the original cylinder, determine all possible values of r.



4. The points $-2 + 2i$, $\frac{-2+2i}{1+i}$, $\frac{-2+2i}{(1+i)^2}$, . . . , $\frac{-2+2i}{(1+i)^8}$ form the consecutive vertices of a polygon in the complex plane. Determine the area of the figure.

5. Find the smallest integer N greater than 10 for which $12N$ has three times as many factors as N.

6. If $.\overline{21}_3 = .\overline{ab}_5$, determine the ordered pair (a, b).

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Answer Sheet

Round 1

1. $a = 44$, b = 7
2. 16
3. 71

Round 2

1. 6.25%
2. 1 : 3
3. $\frac{9}{2}\sqrt{10}$

Round 3

1. 156
2. 5 or 8
3. 11

Round 4

1. $\frac{6m+4p-9n}{6}$
2. $2\sqrt{2}$
3. 4

Round 5

1. $7-3\sqrt{5}$
2. $\frac{1}{2}$
3. $-\frac{\sqrt{3}}{3}$

Round 6

1. $\frac{\pi}{2}$
2. 8
3. $\frac{1}{4}$

Team

1. 504
2. 961
3. $r > 12$
4. $\frac{255}{64}$ or $3\frac{63}{64}$
5. 30
6. (4, 1)