NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2013

Round 1: Arithmetic and Number Theory

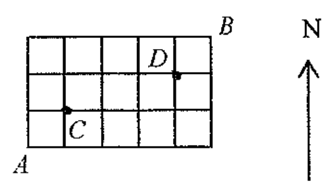
1. ______________________

2. ______________________

3. ______________________

1. A healthy cereal has 5 grams of protein in a 50-gram serving. If the cereal costs \$3.30 for a 16-ounce box, compute the cost per ounce of protein in dollars and cents, rounded to the nearest cent.

2. Going either north or east, how many different routes go from A to B that don't go through points C or D?



3. Given that n is an integer with $1 \le n \le 2013$, for how many values of n does the number $2(n + 3)$ end in 0?

NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2013

Round 2: Algebra 1

1. ___(_____,_____)_____

2. ____________________

3. ____________________

1. If $(x,y)=\left(\frac{2}{3},\frac{3}{5}\right)$ satisfies the system $\begin{matrix} mx-5y=-1 \\ 3x+ny=8 \end{matrix}$, compute the ordered pair $(m,\ n)$

2. Compute all real solutions to $\frac{6x^{-1}+1}{12x^{-1}+2}=\frac{1}{2}$.

3. If 10101 in base b equals 101 in base $2b$ for $b>0$, what is the value of b?

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Round 3: Geometry

1. ____________________

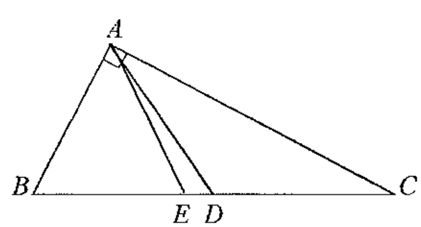
2. ____________________

3. ____________________

1. The measures of two of the angles of a triangle are $x + 40$ and $3x + 10$. All of the angle measures are integers. Determine the smallest possible degree measure of an angle of the triangle.

2. A circle is circumscribed about regular hexagon *ABCDEF*. Rectangle *BCEF* is drawn. Let the area of region I be the total area inside the circle but outside the hexagon. Let the area of region II be the total area outside the rectangle but inside the hexagon. Compute the ratio of the area of region I to the area of region II, given that each side of the hexagon measures 4 units.

3. ΔBAC is a right triangle with $m\angle BAC = 90$, D is the midpoint of $\overline{BC}$, $\overline{AB} \cong \overline{AE}$, $ED = 2$ and $DC = 8$. Compute the value of AB.



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Round 4: Algebra 2

1. ____________________

2. ____________________

3. ___(_____,_____)_______

1. For $a \in \{2,3,4,5\}$ and $b \in \{2,3,4,5\}$, determine the number of distinct values of x such that x is a non-zero root of $ax^2 + bx = x$.

2. If $2013^a = 100$ and $0.2013^b = 100$, compute $\frac{1}{a} - \frac{1}{b}$.

3. Computer the coordinates of the ordered pair (x, y) satisfying the following system:

$$x + \left(\frac{7-\sqrt{51}}{2}\right)y = \left(\frac{7-\sqrt{51}}{2}\right)^2$$

$$x + \left(\frac{7+\sqrt{51}}{2}\right)y = \left(\frac{7+\sqrt{51}}{2}\right)^2$$

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Round 5: Analytic Geometry

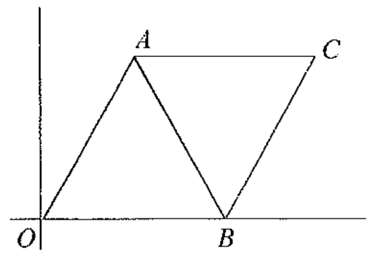
1. ____________________

2. (____ , ____ , ____)

=

3. ____________________

1. ΔAOB and ΔCAB are equilateral triangles. Compute the slope of $\overline{OC}$.



2. An ellipse has one end of its major axis at the y-intercept of $7x - 8y = 32$ and the ends of its minor axis on the ends of the vertical diameter of $(x-3)^2 + (y+4)^2 = 4$. If the equation of the parabola whose vertex is at the lower end of the minor axis of the ellipse and which passes through the ends of the major axis is in the form $y = ax^2 + bx + c$, compute the coordinates of the ordered triple (a, b, c)?

3. The center of circle Q lies in the first quadrant below the graph of $xy = 2$. Circle Q is tangent to the positive x- and y-axes as well as to $xy = 2$. Compute the sum of the coordinates of the center of Q.

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Round 6: Trig and Complex Numbers

1. ____________

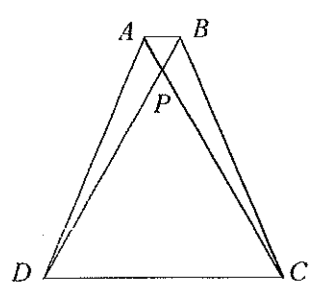
2. ____________

3. ____________

1. Determine the rectangular form of $(4\text{ cis }45°)^2 \cdot (2\text{ cis }150°)$

2. With one end stuck on the ground a telephone pole is raised to the vertical position. If the shadow of the pole loses 20 feet in length as the pole's angle of inclination with the ground increases from 30° to 60°, compute the length of the pole.

3. $ABCD$ is an isosceles trapezoid in which $m\angle CAB = 60°$. If the sum of the areas of ΔAPD and ΔBPC equals 18, compute the value of the product $BP \cdot PC$.



MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

NEW ENGLAND PLAYOFFS – 2013

Team Round

1. ____________________ **4.** ____________________

2. ____________________ **5.** ____________________

3. ____________________ **6.** ____________________

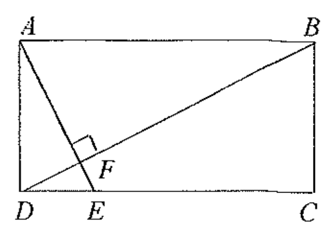
1. For $a \geq 1$ and $c \geq 1$, if $2^a, 4, 2^c$ form an increasing arithmetic sequence, compute the largest possible value of c.

2. Compute the value of $1003^3 - 3 \cdot 1001^3 + 3 \cdot 999^3 - 997^3$.

3. The equation $I = \dfrac{AM}{ME}$ represents a two-digit number being divided by a two-digit number.

   The result is a single digit. If the letters I, A, M, and E represent different non-zero digits, what values can I take on?

4. $ABCD$ is a rectangle, $\overline{AE} \perp \overline{DB}$, the area of ΔDFA is 24, the area of ΔAFB is 72, and area of quadrilateral $BCEF$ is 88. Compute the value of the product $(AE)(DB)$.



5. Compute the area of the convex polygon in the complex plane whose vertices are the complex solutions to $\left(z^2 + \frac{1}{z^2}\right)^2 + \left(z + \frac{1}{z}\right)^2 = 4$.

6. Point A lies on the positive x-axis, B lies on the positive y-axis, and O is the origin. P and Q are trisection points of $\overline{AB}$. If the slope of $\overline{AB}$ is k, find the product of the slopes of $\overline{OP}$ and $\overline{OQ}$ in terms of k.

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Answer Sheet

Round 1

1. \$2.06
2. 12
3. 403

Round 2

1. (3, 10)
2. All reals except 0, –6
3. $\sqrt{3}$

Round 3

1. 1
2. $\frac{2\pi\sqrt{3}-9}{3}$
3. $4\sqrt{3}$

Round 4

1. 13
2. 2
3. $\left(\frac{1}{2}, 7\right)$

Round 5

1. $\frac{\sqrt{3}}{3}$
2. $\left(\frac{2}{9}, -\frac{4}{3}, -4\right)$
3. $4\sqrt{2} - 4$

Round 6

1. $-16 - 16i\sqrt{3}$
2. $20\sqrt{3} + 20$
3. $12\sqrt{3}$

Team

1. $\log_2 6$
2. 48
3. 2, 3, 4, 7
4. 256
5. $\frac{3\sqrt{3}}{2}$
6. k^2