NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2016

Round 1: Arithmetic and Number Theory

1. _____

2. _____

3. _____

1. $4B3C_9$ is a 4-digit base 9 number such that $C = 3B$. What is the base 9 sum of all possible numbers satisfying the given condition?

2. *ABCD* is a four-digit positive integer such that *D* is twice *C*, $D \neq 0$, and *BCD* is a three-digit integer that is twice the three-digit integer *ABC*. Compute all possible ordered quadruples (*A, B, C, D*). (Proper ordered quadruple notation must be used.)

3. Compute the number of positive integers n less than 50 such that $n-3$ and $n+3$ are both prime.

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Round 2: Algebra 1

1. _____

2. _____

3. _____

1. Compute the number of degrees Fahrenheit such that the number given by the Fahrenheit scale for a temperature is twice the number given by the Centigrade scale for the same temperature. (Remember the relationship between Fahrenheit and Celsius is linear with 0°C = 32°F and 100°C = 212°F).

2. In trying to solve an equation of the form $\frac{1}{a}+\frac{2016}{x}=4$, Jean miswrote the equation as $\frac{2016}{ax}=4$, but ended up with the same answer as the original equation. Compute the value of a for which this is possible.

3. Compute <u>all</u> values of x (a real number) for which $\sqrt{\frac{x^2+3}{x}}-\sqrt{\frac{x}{x^2+3}}=\frac{3}{2}$.

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Round 3: Geometry

1. _____

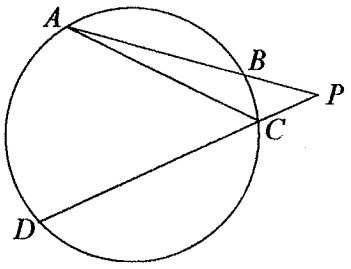
2. _____

3. _____

1. Q lies in the exterior of $\angle ABC$. If $m\angle ABQ = 97$ and $m\angle CBQ = 84$, compute all possible measures of $\angle ABC$ between 0° and 180°.

2. $ABCDEFGH$ is a cube, each edge having a length of 30 units. $ABCD$ is a cube face and $EFGH$ is its opposite face. $\overline{AG}, \overline{BH}, \overline{CE}$ and $\overline{DF}$ are cube diagonals. M is $\frac{1}{3}$ of the way from D to C. N is the midpoint of $\overline{CG}$. $MCNB$ is a pyramid. Determine the number of cubic units in the space of the cube exterior to the pyramid.

3.. $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are secants of a circle and meet outside the circle at P as shown in the diagram. If the degree measure of arc $\overset{\frown}{AD}$ is 8 times the degree measure of $\angle BAC$, compute the largest possible integer value for the measure in degrees of $\angle P$.



NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

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Round 4: Algebra 2

1. _____

2. _____

3. _____

1. Compute <u>all</u> real values of x for which $(\log_6 x)^2 + 3\log_6(6x) - \frac{1}{2}\log_{\sqrt{6}} 6 = 0$

2. If $f(x) = \frac{ax}{x+2}$, $x \neq -2$, compute a so that f is its own inverse.

3. Given a line for which the x-intercept, slope, and y-intercept, taken in this order, form an arithmetic sequence with a common difference of $\frac{15}{2}$. Compute <u>all</u> possible values of the y-intercept.

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Round 5: Analytic Geometry

1. _____

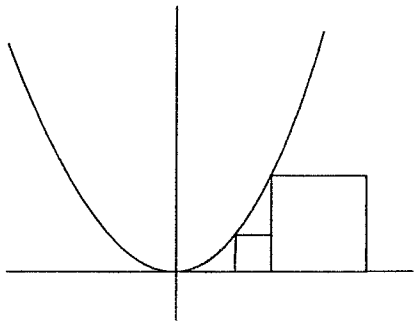
2. _____

3. _____

1. The intersection points of the graphs of $y = 2x^2 - 4x - 1$ and $y = -4x + 7$ determine a line segment. Compute the slope of the perpendicular bisector of that line segment.

2. An ellipse has an area of 100π and an eccentricity of 0.6. Compute the length of a latus rectum in this ellipse. (note: eccentricity of an ellipse is the distance between its center and either of its two foci; The chord through a focus and perpendicular to the major axis of the ellipse is called its latus rectum.)

3. Two squares are placed with a side on the x-axis and a corner on $y = x^2$ as shown. A side of the smaller square lies on a side of the larger. If the ratio of their areas is 81, find the side of the smaller square.



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Round 6: Trig and Complex Numbers

1. _____

2. _____

3. _(_____,_____,_____)

1. For how many positive integral values of n less than 1000 is $\left(\cos\frac{\pi}{7}+i\sin\frac{\pi}{7}\right)^n$ a real number?

2. Compute the value of x if $\tan\left(\sin^{-1}x\right)=2$.

3. In ΔABC, $AB=\cos\angle A$, $AC=\sin\angle A$, and $BC=\frac{1}{2}$. One of the triangles determined by this data is isosceles. The length of $\overline{AC}$ for the non-isosceles triangle can be written as $\frac{a+\sqrt{b}}{c}$. Determine the ordered triple (a,b,c)

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

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Team Round - Place all answers on the team round answer sheet.

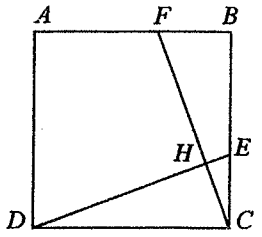
1. Compute all ordered pairs of real numbers (x, y) for which

$$\frac{9x}{2}+\frac{7}{4y}+\frac{9}{16}=0 \text{ and } \frac{4}{3x}=\frac{y}{2}-8$$

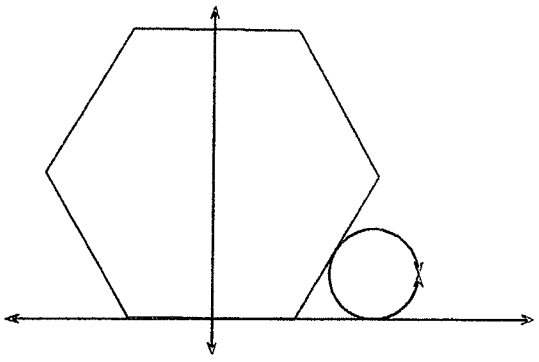
Answers must be in proper ordered pair notation.

2. Let the coordinates of point P be (x, y), where $x<0$ and $y>0$. The distance from point P to the point $Q(3,7)$ is $\sqrt{65}$. Find all possible ordered pairs (x, y) where x and y are integers. Answers must be in proper ordered pair notation.

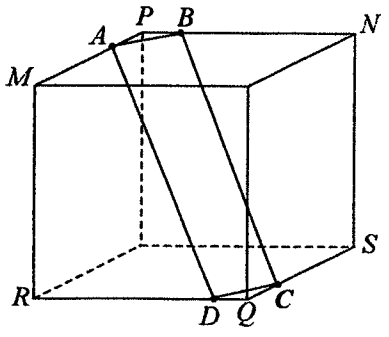
3. $ABCD$ is a square of side 60 units and $BF = CE = 20$. Compute the number of square units in the area of $AFHD$.



4. A circle whose area is 16π square units is tangent to the positive x-axis and to a side of a regular hexagon of side 12 units. The hexagon's center is on the positive y-axis and its bottom side lies on the x-axis. Compute the coordinates of the point of tangency of the circle and the hexagon.



5. In the cube shown, A and B are the trisection points of edges $\overline{MP}$ and $\overline{NP}$, respectively, that are closest to P. C and D are the trisection points on edges $\overline{RQ}$ and $\overline{SQ}$, respectively, that are closest to Q. If the area of $ABCD$ is $\sqrt{17}$, compute the surface area of the cube.



6. Let x and y be integers between 1 and 9 inclusive. Compute the number of ordered pair solutions (x, y) such that $\log_x y + \log_y x^2 = 3$.

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Answer Sheet

Round 1

1 13410_9

2. (1, 2, 4, 8) and (3, 7, 4, 8)

3. 9

Round 2

1. 320

2. $\frac{1}{2}$

3. 1, 3

Round 3

1. 13, 179

2. 25500

3. 107

Round 4

1. $\frac{1}{6}, \frac{1}{36}$

2. –2

3. $\frac{25}{2}, 9$

Round 5

1. $\frac{1}{4}$

2. $\frac{32\sqrt{5}}{5}$

3. 4

Round 6

1. 142

2. $\frac{2\sqrt{5}}{5}$ OR $\pm\frac{2\sqrt{5}}{5}$

3. $(-1, 13, 4)$

Team

1. $\left(-\frac{3}{32}, -\frac{112}{9}\right), \left(-\frac{2}{9}, 4\right)$

2. $(-1, 14), (-4, 3), (-4, 11), (-5, 6), (-5, 8)$

3. 2460

4. ~~$2\sqrt{3}, 6$~~ $(6+2\sqrt{3}, 6)$

5. $27\sqrt{2}$

6. 10