NEW ENGLAND ASSOCIATION OF MATHEMATICS LEAGUES

PLAYOFFS – 2017

Round 1: Arithmetic and Number Theory

1. ____________________

2. ____________________

3. ____________________

1. Reversible primes are prime numbers such that the number with the digits reversed is also a prime number. How many two-digit reversible primes are there?

2. For $0 < p < 100$, compute the number of integer values of p such that $p^3 + 6p^2$ is a perfect square.

3. Compute the remainder when $1 + 4 + 4^2 + 4^3 + \cdots + 4^{2016}$ is divided by 17.

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Round 2: Algebra 1

1. ____________________

2. ____________________

3. ____________________

1. Compute the greatest integer value of k, such that $7x^2 + 20x + k = 0$ has two distinct rational solutions.

2. Simplify completely: $$\frac{4}{y+3} - 2 - \frac{2y}{3-y} + \frac{12y}{9-y^2}$$

3. In a group of boys and girls, each boy pays for a hamburger patty and each girl pays for a roll. (Assume a roll costs x¢ and a hamburger patty costs y¢, where x and y are integers.) They would have spent 10¢ less, if each boy bought a roll and each girl bought a hamburger patty. There are more boys than girls. Compute all possible differences between the number of boys and the number of girls.

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Round 3: Geometry

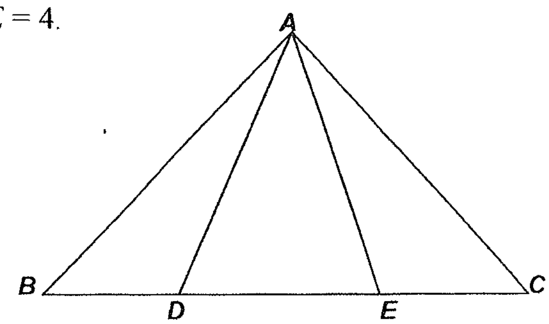
1. ____________________

2. P (_____, _____)

Q (_____, _____)

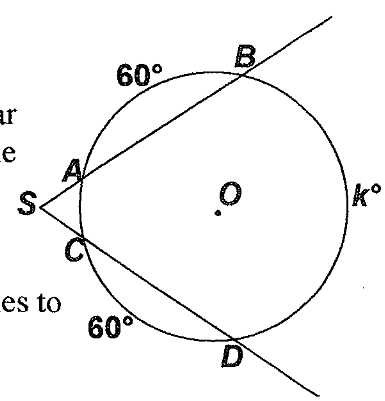
3. ____________________

1. In isosceles triangle ABC, $AB = AC = 12$ and $BD = CE = 4$. If $\text{area}(\Delta BAD) : \text{area}(\Delta ADE) = 4:5$, compute BC.



2. Circle C is tangent to the graphs of $x = 2$, $x = -8$, and $y = -3$. The line $y = \frac{1}{3}x + \frac{14}{3}$ intersects circle C in points P and Q. Compute the coordinates of P and Q, where P is located to the left of Q.

3. For a <u>minimum</u> value of n, the number of diagonals in a regular polygon P with n sides is a multiple of 7, but n is not a multiple of 7. One angle in a scalene triangle has the same measure as an interior angle of P. The measure of the obtuse angle formed by the bisectors of the other two angles in this scalene triangle is $k°$. Compute the $m\angle S$ formed by the two secant lines to circle O in the diagram at the right.



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Round 4: Algebra 2

1. ____________________

2. (______, ______)

3. ____________________

1. Factor completely over the integers: $x^2 4^x + 256 - 2^{2x+2} - 64x^2$

2. In an arithmetic sequence, the sum of the first six terms is 261, and the sum of the first nine terms is 297. The common difference and the first term of this sequence are d and a, respectively. Compute the ordered pair (a, d).

3. The population of a colony of bacteria is given by $P(t) = 100 \cdot \dfrac{3 + 4^t}{1 + 2^t}$. The time t_d that it takes to double in size from $t = 0$ is given by $\log_2 K$. Compute K.

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Round 5: Analytic Geometry

1. ____________________

2. ____________________

3. ____________________

1. Compute the area bounded by the graph of $y = -|2x-5|+6$ and the coordinate axes.

2. The length of line segment $\overline{AB}$ is 18. It moves so that A is on the positive y-axis and B is on the positive x-axis. Point P is on $\overline{AB}$, 6 units from B. There is a position of $\overline{AB}$ such that lines drawn from P perpendicular to the axes form a square whose two other sides are on the axes. Compute the area of that square.

3. Point O has coordinates $(0,0)$ and point A has coordinates $(4,8)$ and point B lies on the $x-$axis. Compute the slopes of all lines through A and B such that the area of ΔAOB is 20.

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Round 6: Trig and Complex Numbers

1. ____________________

2. ____________________

3. (______, ______, ______)

1. If $\csc\alpha = \frac{25}{24}$, where $90° < \alpha < 180°$, and $\cot\beta = \frac{12}{5}$, where $180° < \beta < 270°$, compute $\cos(\alpha + \beta)$.

2. Let $z = 1 + 2i$. Compute the area of a triangle whose vertices in the complex plane are z, $\bar{z}$, and $\frac{1}{z}$.

3. Compute $\sin 2\theta$, if $\theta = \text{Tan}^{-1}\left(\frac{1}{3}\right) + \text{Tan}^{-1}\left(-\frac{1}{2}\right)$

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

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Team Round - Place all answers on the team round answer sheet.

1. $\left(3n^2, 11(n+1), P\right)$ is a Pythagorean Triple, where $(n + 1)$ and P are prime and P is the largest term in the ordered triple. Determine the smallest possible value of P.

2. There are k ordered pairs of relatively prime positive integers (m_i, n_i), where $m_i < n_i$ and $m_i + n_i = 91$. Compute $S = \sum_{i=1}^{i=k} m_i$.

3. The lattice points on the circle $(x-1)^2 + (y-5)^2 = 8$ determine a square. An ellipse defined by $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, where $a > b$, passes through these 4 points. The foci of the ellipse, each c units from the center, lie on two opposite sides of the square determined by these 4 points. $(abc)^2$, in the simplified form, may be expressed as $c_1\left(c_2 + \sqrt{c_3}\right)$, for integers $c_1, c_2,$ and c_3. Determine the ordered triple (c_1, c_2, c_3).

4. Given: Two concentric circles, where the radius of the inner circle is r and the ring between them has width a. Two perpendicular chords $\overline{AB}$ and $\overline{CD}$ are tangent to the inner circle and intersect at point P, where $PC < PD$. Let $PC = b$, where $b < a$ and $a \neq 2b$. For a minimum value of $a + b$, r is rational. Compute the ordered triple (a, b, r).

5. The probability of a weighted coin landing heads up is p. The probability of getting at least 4 heads in 6 tosses is greater than 50%. Determine p, rounded to the nearest 0.1.
   ↑ THE SMALLEST POSSIBLE

6. In ΔPQR, $\sin P : \sin Q : \sin R = 2:3:4$ and $\cos P : \cos Q : \cos R = a:b:c$, where $a > 0$. Compute the ordered triple (a,b,c).

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Answer Sheet

Round 1

1. 9
2. 8
3. 1

Round 2

1. 13
2. $-\frac{2}{y+3}$
3. 1, 2, 5, 10

Round 3

1. 13
2. (–8, 2) and (1, 5).
3. 42

Round 4

1. $(2^x+8)(2^x-8)(x+2)(x-2)$
2. $(61,-7)$
3. $2+\sqrt{5}$

Round 5

1. $\frac{71}{4}$
2. $\frac{144}{5}$
3. $-8, \frac{8}{9}$

Round 6

1. $\frac{204}{325}$
2. $\frac{8}{5}$
3. $-\frac{7}{25}$

Team

1. 73
2. 810
3. $(64,2,5)$
4. $(8,7,5)$
5. 0.6
6. $(14,11,-4)$