2019 CT ARML Runoff, Part I		60 minutes, NO CALCULATOR
Please use a black or blue pen an	nd write legibly.	
Name:	School:	Grade:

- Please write your final answers in the boxes on the right. Problems are weighted equally.
- You do not need to show your work, and there is no penalty for guessing.
- This sheet will be scanned. Please use a black or blue pen and write legibly.
- No aids are permitted other than blank paper, graph paper, ruler, compass and protractor.
- You may not use a calculator nor any electronics. You have 60 minutes for 12 problems.

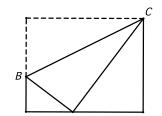
1.	Robert is 6 years older than Jessica. Six years ago he was twice as old as she was. How old is Jessica now?	12
2.	Find the base $b$ of the numeration system in which $232_b = 2_b \times 114_b$	6
3.	There are four identical slices of cheese pizza and four identical slices of pepperoni pizza. Anne, Alex, Abe, and Alice will share the slices of pizza. In how many distinguishable ways can the eight slices of pizza be distributed if each person receives exactly two slices?	19
4.	A ladder is leaning against a house with the bottom 15 feet from the house. When its bottom is pulled 9 feet farther away from the house, the upper end slides 13 feet down.  How many feet long is the ladder?	25
5.	Find the value(s) of $x$ , if the reciprocal of $x + 1$ is $x - 1$ .	$\pm\sqrt{2}$
6.	A small car radiator has a 6-liter capacity and it is filled with liquid which is 40% antifreeze. How many liters of the liquid must be replaced with pure antifreeze so that the radiator becomes filled with a mixture containing 60% antifreeze?	2

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7.	What is the sum of the solutions of $ x + 3  = 3 x - 2 $ ?

 $\frac{21}{4}$ 



 $10\sqrt{5}$ 

9. How many positive integers are divisors of 
$$23^4 + 4 \times 23^3 + 6 \times 23^2 + 4 \times 23 + 1$$
?

65

10. Suppose 
$$f$$
 is a function such that

$$f(x) + f(1 - x) = 10$$
 and  $f(x + 1) = 4 + f(x)$ .

6

11. What is the value of b if a, b, and c satisfy the foll

What is the value of f(100) + f(-100)?

11. What is the value of b if a, b, and c satisfy the following equations?

$$1 + a + b + c = 16 + 8a + 4b + 2c =$$

35

$$= 81 + 27a + 9b + 3c = 256 + 64a + 16b + 4c$$

12. In DABC we have

$$3\sin A + 4\cos B = 6$$
 and  $4\sin B + 3\cos A = 1$ .

30

What is the degree measure of DC?

2019 CT ARML Run	off, Part II	60 minutes, NO CALCULATOR
Please use a black or b	<b>lue pen</b> and write legibly.	
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- You do not need to show your work, and there is no penalty for guessing.
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- You may not use a calculator nor any electronics. You have 60 minutes for 12 problems.

13.	Find the ordered pair $(x,y)$ such that	
	$xy$ , $\frac{x}{y}$ , and $x - y$ are all equal.	$\left(-\frac{1}{2},-1\right)$
14.	In a parallelogram $ABCD$ , $AB = CD = 8$ .  A line intersecting side $\overline{AB}$ at $E$ and side $\overline{CD}$ at $F$ cuts the parallelogram into two polygons of equal areas.  If $AE = 3$ , find length $DF$ .	5
15.	What is the probability that a randomly chosen divisor of 2025 is divisible by 5?	$\frac{2}{3}$
16.	Consider the equations $x^2 + kx + 6 = 0$ and $x^2 - kx + 6 = 0$ . When the roots of the equations are suitably listed, each root of the second equation is 5 more than the corresponding root of the first equation. What is the value of $k$ ?	5
17.	Two circles whose equations are $x^2 + y^2 = 25$ and $x^2 + y^2 - 2x + 6y - 15 = 0$ intersect at A and B.  Compute the slope of line AB.	$\frac{1}{3}$
18.	A square has its base on the <i>x</i> -axis and one vertex on each branch of the curve $y = 1/x^2$ .  What is the area of the square?	$2^{4/3}$ or $2\sqrt[3]{2}$

Please use a black or blue pen and write legit	oly.
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Name: \_\_\_\_\_ School: \_\_\_\_\_ Grade: \_\_\_\_\_

19.	A rhombus of side length <i>s</i> has the property that there is a point on its longest diagonal such that the distances from that point to the vertices are 1, 1, 1, and <i>s</i> . What is the value of <i>s</i> ?	$\frac{\sqrt{5}+1}{2}$
20.	The six solutions of $z^6 = -64$ are written in the form $a + bi$ where $a$ , $b$ are real numbers. What is the product of the solutions with $a > 0$ ?	4
21.	In the diagram on the right, a square is divided into three pieces by making two parallel cuts. The three resulting pieces have equal areas. The middle piece has a height of 10. What is the area of the entire square?	1300
22.	How many positive integers $\leq$ 2019 have strictly more 1's than 0's in their binary expansion?	1171
23.	Let $x = \sqrt{3}/3$ and $(x+1)(y+1) = 2$ . Find $\arctan y$ .	<u>p</u> 12
24.	Find all ordered pairs $(x,y)$ that solve the system $\begin{cases} \log_8 x \cdot \log_{25} y = \frac{4}{3} \\ \log_x 8 + \log_y 125 = \frac{9}{4} \end{cases}$	(4,625) and (16,25)